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17/04/2021

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$$u_t - Cu_x = 0$$

$$\frac{du}{dt} = C \frac{d^2u}{dx^2} = 0$$

$$\frac{du}{dx} = C \frac{d^2u}{dx^2}$$

$$u_{i,j+1} - u_{i,j} = C \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} \Delta t$$

$$u_{i,j+1} - u_{i,j} = \frac{C \Delta t}{\Delta x^2} [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}] \quad \text{Let } \Delta t / \Delta x^2 = r$$

$$u(x, 0) = x^2$$

$$\Delta x = 0.2 \text{ m}, \quad \Delta t = 0.02 \text{ day}$$

for various conditions

$$\text{At } x = 0 \rightarrow 2x^2 = 0$$

$$\text{At } x = 0.2 \rightarrow 2 \times 0.2^2 = 1.6 \times 10^{-2}$$

$$\text{At } x = 0.4 \rightarrow 2 \times 0.4^2 = 0.32$$

$$\text{At } x = 0.6 \rightarrow 2 \times 0.6^2 = 0.72$$

$$\text{At } x = 0.8 \rightarrow 2 \times 0.8^2 = 1.28$$

$$\text{At } x = 1 \rightarrow 2 \times 1^2 = 2$$

$$u_{i,j+1} = u_{i,j} + r [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$$

$$u_{i,j+1} = r u_{i+1,j} + (1-2r) u_{i,j} + r u_{i-1,j}$$

when  $i=1, j=0$

$$u_{1,1} = 0.5 u_{0,0} + 0.5 u_{2,0}$$

$$= 0.5(0) + 0.5(0.32)$$

$$U_{1,1} = 0.0128$$

when  $i=2, j=0$

$$U_{2,1} = 0.5(U_{1,0}) + 0.5(U_{0,0})$$

$$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$U_{2,1} = 0.0656$$

when  $i=3, j=0$   $U_{3,1} = 0.5(U_{2,0}) + 0.5(U_{4,0})$

$$= 0.5(0.0256) + 0.5(0.4076)$$

$$U_{3,1} = 0.2124$$

or

when  $i=4, j=0$   $U_{4,1} = 0.5(U_{3,0}) + 0.5(U_{5,0})$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$U_{4,1} = 0.5698$$

For  $j=1$

when  $i=1$   $\therefore U_{1,2} = 0.5(U_{0,1}) + 0.5(U_{2,1})$

$$= 0.5(U_{0,1}) + 0.5(U_{2,1}) = 0 + 0.5(0.0656)$$

$$U_{1,2} = 0.0328$$

when  $i=2$   $\therefore U_{2,2} = 0.5(U_{1,1}) + 0.5(U_{3,1})$

$$= 0.5(0.0328) + 0.5(0.2124)$$

$$U_{2,2} = 0.1152$$

when  $i=3$   $U_{3,2} = 0.5(U_{2,1}) + 0.5(U_{4,1})$

$$= 0.5(0.0656) + 0.5(0.5698)$$

$$U_{3,2} = 0.3152$$

$$\begin{aligned} \text{When } i=4 \quad U_{4,2} &= 0.5(U_{2,1}) + 0.5(U_{5,1}) \\ &= 0.5(0.2126) + 0.5(1) \\ U_{4,2} &= 0.4088 \end{aligned}$$

For  $J=2$

$$\begin{aligned} \text{When } i=1 \quad \therefore U_{1,3} &= 0.5(U_{0,1}) + 0.5(U_{2,2}) \\ &= 0 + 0.5(0.1152) = 0.0576 \end{aligned}$$

$$\begin{aligned} \text{When } i=2 \quad U_{2,3} &= 0.5(U_{1,2}) + 0.5(U_{3,2}) \\ &= 0.5(0.0328) + 0.5(0.2152) = 0.124 \end{aligned}$$

$$\begin{aligned} \text{When } i=3 \quad U_{3,3} &= 0.5(U_{2,2}) + 0.5(U_{4,2}) \\ &= 0.5(0.1152) + 0.5(0.6088) = 0.362 \end{aligned}$$

$$\begin{aligned} \text{When } i=4 \quad U_{4,3} &= 0.5(U_{3,2}) + 0.5(U_{4,2}) \\ &= 0.5(0.3152) + 0.5(1) \\ U_{4,3} &= 0.6576 \end{aligned}$$

For  $J=3$

$$\begin{aligned} \text{When } i=1 \quad U_{1,4} &= 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ &= 0.5(0) + 0.5(0.134) = 0.084 \end{aligned}$$

$$\begin{aligned} \text{When } i=2 \quad U_{2,4} &= 0.5(U_{1,3}) + 0.5(U_{3,3}) \\ &= 0.5(0.0576) + 0.5(0.362) = 0.2078 \end{aligned}$$

$$\begin{aligned} \text{When } i=3 \quad U_{3,4} &= 0.5(U_{2,3}) + 0.5(U_{4,3}) \\ &= 0.5(0.124) + 0.5(0.6576) = 0.4158 \end{aligned}$$

$$\begin{aligned} \text{When } i=4 \quad U_{4,4} &= 0.5(U_{3,3}) + 0.5(U_{5,3}) \\ &= 0.5(0.362) + 0.5(1) = 0.681 \end{aligned}$$

For  $J=4$

$$\text{when } i=1 \quad u_{1,5} = 0.5(u_{1,4}) + 0.5(u_{2,4})$$

$$= 0.5(0.2098) + 0.5(0.1099) = 0.1099$$

$$\text{when } i=2 \quad u_{2,5} = 0.5(u_{1,4}) + 0.5(u_{3,4})$$

$$= 0.5(0.087) + 0.5(0.4158) = 0.2514$$

$$\text{when } i=3 \quad u_{3,5} = 0.5(u_{2,4}) + 0.5(u_{4,4})$$

$$= 0.5(0.2098) + 0.5(0.681) = 0.4454$$

$$\text{when } i=4 \quad u_{4,5} = 0.5(u_{3,4}) + 0.5(u_{5,4})$$

$$= 0.5(0.4158) + 0.5(1)$$

$$= 0.7079$$

$\Delta t$	$J$ Temp(K)						
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1
0.08	4	0	0.087	0.2098	0.4158	0.681	1
0.06	3	0	0.596	0.174	0.362	0.6524	1
0.04	2	0	0.328	0.1152	0.3152	0.6088	1
0.02	1	0	0.0128	0.0656	0.2126	0.5648	1
0	0	0	0.0016	0.028	0.1296	0.4094	1
$\Delta x$		0	0.2	0.4	0.6	0.8	1
	$i$	0	1	2	3	4	5

