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CIVIL ENGINEERING ENG 382

$$u - (u_{xx} = 0)$$

$$\frac{du}{dt} = \frac{C d^2 u}{dx^2} = 0$$

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$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = \frac{C}{\Delta x^2} [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$$

$$u_{i,j+1} - u_{i,j} = \frac{C \Delta t}{\Delta x^2} [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$$

$$u(x,0) = x^4 + k$$

$$\Delta x = 0.2, \quad \Delta t = 0.02 \text{ day}$$

For initial conditions

$$\text{At } x=0 \rightarrow x^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$\therefore u_{i,j+1} = u_{i,j} + r (u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

$$u_{i,j+1} = r u_{i-1,j} + (1-2r) u_{i,j} + r u_{i+1,j}$$

$$u_{i,j+1} = r u_{i-1,j} + (1-2r) u_{i,j} + r u_{i+1,j}$$

when  $i=1, j=0$

$$u_{1,1} = 0.5 u_{0,0} + 0.5 u_{2,0}$$

$$= 0.5(0) + 0.5(0.0256)$$

$$u_{1,1} = 0.0128$$

when  $i=2, j=0$

$$u_{2,1} = 0.5(u_{1,0}) + 0.5(u_{3,0})$$

$$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$u_{2,1} = 0.0656$$

$$\begin{aligned} \text{when } i=3, j=0 & \Rightarrow u_{3,1} = 0.5(u_{2,0}) + 0.5(u_{4,1}) \\ & = 0.5(0.0256) + 0.5(0.4096) \\ u_{3,1} & = 0.2176 \end{aligned}$$

$$\begin{aligned} \text{when } i=4, j=0 & \Rightarrow u_{4,1} = 0.5(u_{3,0}) + 0.5(u_{5,1}) \\ & = 0.5(0.0256) + 0.5(0.4096) \\ u_{4,1} & = 0.5648 \end{aligned}$$

For j=1

$$\begin{aligned} \text{when } i=1 & \dots u_{1,2} = 0.5(u_{1,1}) + 0.5(u_{2,1}) \\ & = 0.5(0) + 0.5(0.0656) \\ u_{1,2} & = 0.328 \end{aligned}$$

$$\begin{aligned} \text{when } i=2 & \dots u_{2,2} = 0.5(u_{1,1}) + 0.5(u_{3,1}) \\ & = 0.5(0.0256) + 0.5(0.2176) \\ u_{2,2} & = 0.1152 \end{aligned}$$

$$\begin{aligned} \text{when } i=3 & \dots u_{3,2} = 0.5(u_{2,1}) + 0.5(u_{4,1}) \\ & = 0.5(0.2176) + 0.5(0.5648) \\ u_{3,2} & = 0.3152 \end{aligned}$$

$$\begin{aligned} \text{when } i=4 & \dots u_{4,2} = 0.5(u_{3,1}) + 0.5(u_{5,1}) \\ & = 0.5(0.2176) + 0.5(1) \\ u_{4,2} & = 0.6088 \end{aligned}$$

$$= 0.5(0.152) + 0.5(0.6088) = 0.362$$

$$\text{when } i=4 = 26413 = 0.5(21313) + 0.5(21512)$$

$$2 = 0.5(0.3152) + 0.5(1)$$

$$26413 = 0.6576$$

When J=3

$$\text{when } i=1 \quad 26 = 0.5(0) + 0.5(0.174) = 0.087$$

$$\text{when } i=2 \quad = 0.5(0.0576) + 0.5(0.362) = 0.2098$$

$$\text{when } i=3 \quad 21314 = 0.5(21213) + 0.5(21413)$$

$$= 0.5(0.174) + 0.5(0.6576) = 0.4158$$

$$\text{when } i=4 \quad 26414 = 0.5(21313) + 0.5(21513)$$

$$= 0.5(0.362) + 0.5(1) = 0.681$$

When J=4

$$\text{when } i=1 \quad 2115 = 0.5(2104) + 0.5(21214)$$

$$= 0.5(0.087) + 0.5(0.2098) = 0.1049$$

$$\text{when } i=2 \quad 21215 = 0.5(2114) + 0.5(21314)$$

$$= 0.5(0.087) + 0.5(0.4158) = 0.2514$$

$$\text{when } i=3 \quad 21315 = 0.5(2124) + 0.5(21414)$$

$$= 0.5(0.2098) + 0.5(0.681)$$

$$= 0.4454$$

$$\text{when } i=4 \quad 21415 = 0.5(21314) + 0.5(21514)$$

$$= 0.5(0.4158) + 0.5(1)$$

$$= 0.7079$$

$\Delta z$	$J$		$T_{\text{temp}}(K)$				
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1
0.08	4	0	0.087	0.2098	0.4158	0.681	1
0.06	3	0	0.0576	0.174	0.362	0.6576	1
0.04	2	0	0.0328	0.1152	0.3152	0.6088	1

