

Using Explicit forward difference method.

$U_{t+1} = U_{t,x}$ for $0 \leq x \leq 1m$, $0 \leq t \leq 0.1$ day, $\Delta x = 0.2m$

$$x_i = i \cdot \Delta x$$

$$(i,j) \quad i=1,2,3,4,5,6,7,8,9,10$$

$$U_t = C U_{xx}$$

$$C = 1/10$$

$$1 \times 10^{-3}$$

$$U_{t+1} = U_t + C(U_{t,i+1} - U_{t,i-1})$$

$$\text{Initial condition: } U(x,0) = (0.02x^2 + 0.05x + 0.01)(e^{-0.01x}) + 0.001$$

$$U(x,0) = x^2 \kappa + f(x)$$

$$\text{diffusion:}$$

$$0.001 \cdot 10^{-3}$$

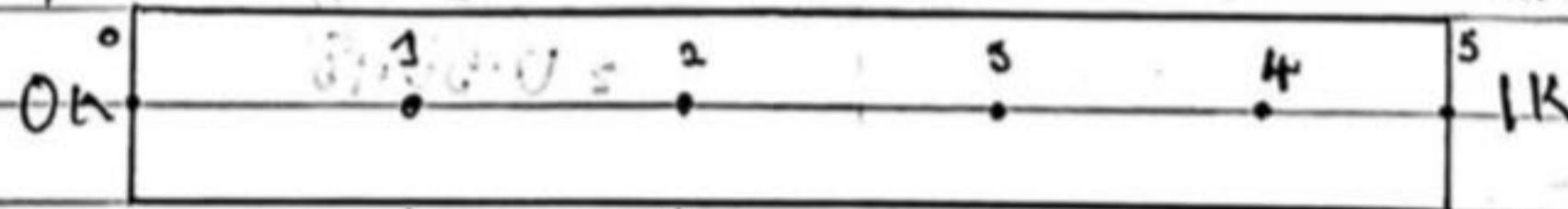
$$0.001 \cdot 0.001$$

Boundary Condition

$$U(0,t) = 0K \rightarrow U(1,t) = 1K$$

$$\text{Graphically: } U(x,0) = (0.02x^2 + 0.05x + 0.01)e^{-0.01x}$$

$$i=0, j=0 \therefore t=0$$



Since $U = 0$ at Boundary condition, we use the function of x .
At $t=0$

$$U_{0,0} = 0^{\circ}K$$

When $x=0.4m$

When $x=0.8$

$$U_{1,0} = (0.4)^4$$

$$U_{1,0} = (0.8)^4$$

When $x=0.2m$

$$= 0.0256$$

$$= 0.4096$$

$$U_{1,0} = (0.2)^4$$

$$(0.2)^4 = 0.0256$$

When $x=1$

$$= 0.0016$$

When $x=0.6m$

$$U_{3,0} = (0.6)^4$$

$$U_{3,0} = (1)^4$$

$$= 0.1296$$

= 1

To find the temperature within the condition gradient, we use the discretized forward Euler method.

$$U_{i,j+1} = rU_{i-1,j} + (1-2r)U_{i,j} + rU_{i+1,j}$$

where

$$r = \frac{C \Delta t}{\Delta x^2} = \frac{1 \times 0.02}{(0.2)^2} = 0.5$$

Evaluating U at $t = 0.02 + e^j \Delta t$, $j = 0$, $i = 1, 2, 3, 4$ and $\Delta t = 1$

$$U_{0,0} = 0 \text{ (BC)}$$

$$U_{5,0} = 1 \text{ K}$$

at $i = 1$

$$\begin{aligned} U_{1,1} &= rU_{0,0} + (1-2r)U_{0,0} + rU_{2,0} \\ &= 0.5(0) + (1-2(0.5))(0.0016) + 0.5(0.0008) \\ &= 0 + 0 + 0.0008 \\ &= 0.0008 \end{aligned}$$

at $i = 3$

$$\begin{aligned} U_{3,1} &= rU_{2,0} + (1-2r)U_{3,0} + rU_{4,0} \\ &= 0.5(0.0016) + (1-2(0.5))(0.0008) + 0.5(0.0004) \\ &= 0.0008 + 0.0004 \\ &= 0.0012 \end{aligned}$$

at $i = 2$

$$\begin{aligned} U_{2,1} &= rU_{1,0} + (1-2r)U_{2,0} + rU_{3,0} \\ &= 0.5(0.0008) + (0)(0.0016) + 0.5(0.0004) \\ &= 0.0004 \end{aligned}$$

at $i = 4$

$$\begin{aligned} U_{4,1} &= rU_{3,0} + (1-2r)U_{4,0} + rU_{5,0} \\ &= 0.5(0.0004) + (1-2(0.5))(0.0008) + 0.5(1) \\ &= 0.0002 + 0.0008 \\ &= 0.0010 \end{aligned}$$

$$\therefore U_{1,1}, U_{2,1}, U_{3,1}, U_{4,1} = 0.0008, 0.0004, 0.0012, 0.0010$$

Evaluating U at $t = 0.04$, i.e. $j = 1$

at $i = 1$

$$\begin{aligned} U_{1,2} &= rU_{0,1} + (1-2r)U_{1,1} + rU_{2,1} \\ &= 0.5(0) + (0)(0.0008) + 0.5(0.0004) \\ &= 0.0002 \end{aligned}$$

at $i = 2$

$$\begin{aligned} U_{2,2} &= rU_{1,1} + (1-2r)U_{2,1} + rU_{3,1} \\ &= 0.5(0.0004) + (0)(0.0008) + 0.5(0.0012) \\ &= 0.0002 \end{aligned}$$

at $i=3$

$$U_{3,2} = rU_{2,1} + (1-2r)U_{3,1} + rU_{4,1}$$

$$= 0.5(0.0596) + 0(0.2176) + 0.5(0.5648)$$

$$= 0.3152$$

at $i=4$

$$U_{4,2} = rU_{3,1} + (1-2r)U_{4,1} + rU_{5,1}$$

$$= 0.5(0.2176) + 0(0.5648) + 0.5$$

$$(1)$$

$$= 0.6088$$

N.B:- $U_{0,2} = 0 \neq U_{5,2} = 1$ (due to boundary condition)

Evaluating U at $t=0.06$ i.e $j=2$

at $i=1$

$$U_{1,3} = rU_{0,2} + (1-2r)U_{1,2} + rU_{2,2}$$

$$= 0.5(0) + 0(0.0328) + 0.5$$

$$(0.1152)$$

$$= 0.0596$$

at $i=3$

$$U_{3,3} = rU_{2,2} + (1-2r)U_{3,2} + rU_{4,2}$$

$$= 0.5(0.1152) + 0(0.3152) + 0.5$$

$$(0.6088)$$

$$= 0.3620$$

at $i=2$

$$U_{2,3} = rU_{1,2} + (1-2r)U_{2,2} + rU_{3,2}$$

$$= 0.5(0.0328) + 0(0.1152) + 0.5$$

$$(0.3152)$$

$$= 0.1740$$

at $i=4$

$$U_{4,3} = rU_{3,2} + (1-2r)U_{4,2} + rU_{5,2}$$

$$= 0.5(0.3152) + 0(0.6088) + 0.5$$

$$(1)$$

$$= 0.6596$$

N.B:- $U_{0,3} = 0 \neq U_{5,3} = 1$

Evaluating U at $t=0.08$ i.e $j=3$

Note:- $U_{0,4} = 0 \neq U_{5,4} = 1$

at $i=1$

$$U_{1,4} = rU_{0,3} + (1-2r)U_{1,3} + rU_{2,3}$$

$$= 0.5(0) + 0(0.0596) + 0.5$$

$$(0.1740)$$

$$= 0.087$$

at $i=2$

$$U_{2,4} = rU_{1,3} + (1-2r)U_{2,3} + rU_{3,3}$$

$$= 0.5(0.0596) + 0(0.1740) + 0.5$$

$$(0.362)$$

$$= 0.2098$$

at $i=3$

$$\begin{aligned} U_{3,n} &= rU_{2,3} + (1-2r)U_{3,3} + rU_{4,3} \\ &= 0.5(0.194) + 0(0.362) + 0.5 \\ &\quad (0.6576) \\ &= 0.4158 \end{aligned}$$

at $i=4$

$$\begin{aligned} U_{4,4} &= rU_{3,3} + (1-2r)U_{4,3} + rU_{5,3} \\ &= 0.5(0.362) + 0(0.6576) + 0.5 \\ &\quad (0.681) \\ &= 0.681 \end{aligned}$$

Evaluating U at $t=1, i=4$ at $i=1$

$$\begin{aligned} U_{1,5} &= rU_{0,5} + (1-2r)U_{1,4} + rU_{2,4} \\ &= 0.5(0) + 0(0.087) + 0.5 \\ &\quad (0.2098) \\ &= 0.1049 \end{aligned}$$

at $i=3$

$$\begin{aligned} U_{3,5} &= rU_{2,4} + (1-2r)U_{3,4} + rU_{4,4} \\ &= 0.5(0.2098) + 0(0.4158) + 0.5 \\ &\quad (0.681) \\ &= 0.4454 \end{aligned}$$

at $i=2$

$$\begin{aligned} U_{2,5} &= rU_{1,4} + (1-2r)U_{2,4} + rU_{3,4} \\ &= 0.5(0.087) + 0(0.2098) \\ &\quad + 0.5(0.4158) \\ &= 0.0435 + 0 + 0.2099 \\ &= 0.2514 \end{aligned}$$

at $i=4$

$$\begin{aligned} U_{4,5} &= rU_{3,4} + (1-2r)U_{4,4} + rU_{5,4} \\ &= 0.5(0.4158) + 0(0.681) + 0.5 \\ &\quad (0.681) \\ &= 0.9079 \end{aligned}$$

N/B: $U_{0,5} = 0$ & $U_{5,5} = 1$

t	x	0	0.2	0.4	0.6	0.8	1
0	0	0	0.0016	0.0256	0.1296	0.4096	1
0.02	0.02	0.0016	0.0128	0.0656	0.2196	0.5648	1
0.04	0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.06	0.06	0	0.0576	0.1944	0.362	0.6576	1
0.08	0.08	0	0.087	0.2098	0.4158	0.681	1
0.1	0	0.1049	0.2514	0.4454	0.7079	1	