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Department: ~~Electrical Engineering~~ Civil Engineering

Course Code: ECE 302

$$U_t - C U_{xx} = 0$$

$$\frac{d_t}{dt} - \frac{C d^2}{dx^2} = 0$$

$$\frac{d_t}{dt} = \frac{C d^2}{dx^2} U$$

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta t} = \frac{C \cdot U_i + U_j - 2U_{i,j}}{\Delta x^2}$$

$$U_{i,j+1} - U_{i,j} = \frac{C \Delta t}{\Delta x^2} [U_i + U_j - 2U_{i,j}]$$

$$U(x=0) = x^4 K$$

$$\Delta x = 0.2 \text{ m}, \Delta t = 0.02 \text{ day}$$

for initial conditions

$$\text{At } x=0 \rightarrow x^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$\therefore U_{i,j+1} = U_{i,j} + r [U_i + U_j - 2U_{i,j}]$$

$$U_{i,j+1} = r [U_i + U_j] + (1-2r) U_{i,j} + r [U_i + U_j]$$

when  $i=1, j=0$

$$U_{1,0} = 0.5U_{1,0} + 0.5U_{2,0}$$

$$= 0.5(0) + 0.5(0.4358)$$

$$U_{1,0} = 0.2179$$

when  $i=2, j=0$

$$U_{2,0} = 0.5(U_{1,0}) + 0.5(U_{3,0})$$

$$= 0.5(0.2179) + 0.5(0.1296)$$

$$U_{2,0} = 0.17375$$

when  $i=3, j=0$

$$U_{3,0} = 0.5(U_{2,0}) + 0.5(U_{4,0})$$

$$= 0.5(0.17375) + 0.5(0.4576)$$

$$U_{3,0} = 0.315675$$

when  $i=4, j=0$

$$U_{4,0} = 0.5(U_{3,0}) + 0.5(U_{5,0})$$

$$= 0.5(0.315675) + 0.5(0.4096)$$

$$U_{4,0} = 0.3626375$$

for  $j=1$

when  $i=1$

$$U_{1,1} = 0.5(0) + 0.5(U_{2,1})$$

$$= 0.5(0) + 0.5(U_{2,1})$$

$$= 0.2500$$

$$U_{1,1} = 0.2500$$

when  $i=2$

$$U_{2,1} = 0.5(U_{1,1}) + 0.5(U_{3,1})$$

$$= 0.5(0.2500) + 0.5(U_{3,1})$$

$$= 0.1250 + 0.2500$$

$$U_{2,1} = 0.3750$$

$$\text{when } i=4 \quad U_{4,2} = 0.5(U_{4,1}) + 0.5(U_{4,3})$$

$$= 0.5(0.3176) + 0.5(1)$$

$$U_{4,2} = 0.6088$$

for  $J=2$

$$\text{when } i=1 \quad U_{1,1} = 0.5(U_{1,1}) + 0.5(U_{1,2})$$

$$= 0 + 0.5(0.1152) = 0.0576$$

$$\text{when } i=2 \quad U_{2,2} = 0.5(U_{2,1}) + 0.5(U_{2,3})$$

$$= 0.5(0.0576) + 0.5(0.3072)$$

$$= 0.1744$$

$$\text{when } i=3 \quad U_{3,2} = 0.5(U_{3,1}) + 0.5(U_{3,3})$$

$$= 0.5(0.1152) + 0.5(0.6088)$$

$$= 0.362$$

$$\text{when } i=4 \quad U_{4,2} = 0.5(U_{4,1}) + 0.5(U_{4,3})$$

$$= 0.5(0.3176) + 0.5(1)$$

$$U_{4,2} = 0.6576$$

for  $J=3$

$$\text{when } i=1 \quad U_{1,4} = 0.5(U_{1,3}) + 0.5(U_{1,5})$$

$$= 0.5(0) + 0.5(0.1744)$$

$$= 0.0872$$

$$\text{when } i=2 \quad U_{2,4} = 0.5(U_{2,3}) + 0.5(U_{2,5})$$

$$= 0.5(0.0872) + 0.5(0.362)$$

$$= 0.2246$$

$$\text{when } i=3 \quad U_{3,4} = 0.5(U_{3,3}) + 0.5(U_{3,5})$$

$$= 0.5(0.1744) + 0.5(0.6576)$$

$$= 0.416$$

$$\text{for } i=4 \quad U_{4,4} = 0.5(U_{3,4}) + 0.5(U_{5,2}) \\ = 0.5(0.362) + 0.5(1) \\ = 0.681$$

$$\text{for } j=4 \\ \text{when } i=1 \quad U_{1,5} = 0.5(U_{1,4}) + 0.5(U_{2,4}) \\ = 0.5(0.2098) \\ = 0.1049$$

$$\text{when } i=2 \quad U_{2,5} = 0.5(U_{1,4}) + 0.5(U_{3,4}) \\ = 0.5(0.087) + 0.5(0.4158) \\ = 0.2514$$

$$\text{when } i=3 \quad U_{3,5} = 0.5(U_{2,4}) + 0.5(U_{4,4}) \\ = 0.5(0.2098) + 0.5(0.681) \\ = 0.4454$$

$$\text{when } i=4 \quad U_{4,5} = 0.5(U_{3,4}) + 0.5(U_{5,4}) \\ = 0.5(0.4158) + 0.5(1) \\ = 0.7079$$

$\Delta t$	$j$	$i$					
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1
0.08	4	0	0.087	0.2098	0.4158	0.681	1
0.06	3	0	0.0576	0.174	0.362	0.652	2
0.04	2	0	0.0328	0.1152	0.3152	0.602	1
0.02	1	0	0.0128	0.0576	0.2176	0.502	1
0	0	0	0.0016	0.0288	0.1276	0.402	2
$\Delta x$		0	0.2	0.4	0.6	0.8	1
		$i$	1	2	3	4	5

0.1	0	0.1049	0.2514	0.4454	0.7079	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0.0016	0.0256	0.1296	0.4096	1
	0	0.2	0.4	0.6	0.8	1

0.5

