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COMPUTER ENGINEERING
18/ENG 02/105

ENG 382 ASSIGNMENT 4

Using Explicit forward difference method

$u_t = C u_{xx}$ for $0 \leq x \leq 1m$; $0 \leq t \leq 0.1$ day $\Delta x = 0.2m$

$\therefore u_t = C u_{xx}$

Initial Condition

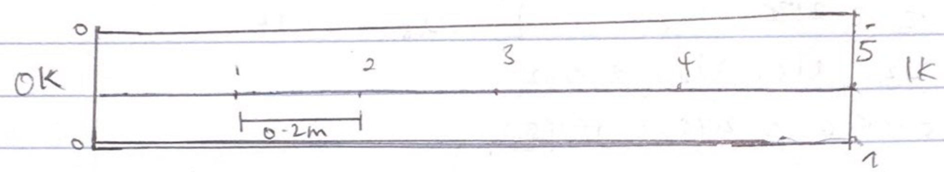
$u(x,0) = x^4 K = f(x)$

Boundary Condition

$u(0,t) = 0K \longrightarrow u(1,t) = 1K$

Graphically

$i = 0, j = 0 \therefore t = 0$



Since it is at Boundary Condition, we will use the function of x at $t = 0$

$u_{0,0} = 0^\circ_k$

When $x = 0.2m$

$u_{1,0} = (0.2)^4 = 0.0016 m$

When $x = 0.4m$

$u_{2,0} = (0.4)^4$

$= 0.0256$

When $x = 0.8$

$u_{4,0} = (0.8)^4$

$= 0.4016$

When $x = 0.6m$

$u_{3,0} = (0.6)^4$

$= 0.1296$

When $x = 1$

$u_{5,1} = (1)^1$

$= 1$

We then use the discretized forward Euler method to find the temperature within the condition gradient.

$u_{i,j+1} = r u_{i-1,j} + (1-2r) u_{i,j} + r u_{i+1,j}$

Where

$r = \frac{C\Delta t}{\Delta x^2} = \frac{1 \times 0.02}{(0.2)^2} = 0.5$

u at $t = 0.02$ ~~for~~ i.e $j = 0, i = 1, 2, 3, 4$

$U_{0,1} = 0$ (BC)

at $i = 2$

$U_{2,1} = rU_{1,0} + (1-2r)U_{2,0} + rU_{3,0}$

$= 0.5(0.0016) + (0)(0.0256) + 0.5(0.126)$

$= 0.0656$

at $i = 1$

$U_{1,1} = rU_{0,0} + (1-2r)U_{1,0} + rU_{2,0}$

$0.5(0) + (1-2(0.5))0.0016 + 0.5(0.0256)$

$= 0 + 0 + 0.128$

$= 0.0128$

at $i = 3$

$U_{3,1} = rU_{2,0} + (1-2r)U_{3,0} + rU_{4,0}$

$= 0.5(0.0256) + (1-2(0.5))(0.1296) + 0.5(0.4096)$

$= 0.2176$

at $i = 4$

$U_{4,1} = rU_{3,0} + (1-2r)(U_{4,0} + rU_{5,0})$

$0.5(0.1296) + (1-2(0.5))(0.4096) + 0.5(1)$ $= 0.5648$

$U_{5,1} = 1$

$\therefore U_{1,1}, U_{2,1}, U_{3,1}, U_{4,1} =$

$= 0.0128, 0.0656, 0.2176, 0.5648$

Evaluating U at t $t = 0.01$ i.e $j = 1$

at $i = 1$

$U_{1,2} = rU_{0,1} + (1-2r)U_{1,1} + rU_{2,1}$

$= 0.5(0) + (0)(0.0128) + 0.5(0.0656)$

at $i = 2$

$U_{2,2} = rU_{1,1} + (1-2r)U_{2,1} + rU_{3,1}$

$= 0.5(0.0128) + (0)(0.0656) + 0.5(0.2176)$

$= 0.1152$

at $i = 3$

$U_{3,2} = rU_{2,1} + (1-2r)U_{3,1} + rU_{4,1}$

$= 0.5(0.0656) + 0(0.2176) + 0.5(0.5648)$

$= 0.3152$

at $i = 4$

$U_{4,2} = rU_{3,1} + (1-2r)U_{4,1} + rU_{5,1}$

$= 0.5(0.426) + 0(0.5648) + 0.5(1)$

$= 0.6088$

NB $U_{0,2} = 0$ & $U_{5,2} = 1$ (due to boundary condition)

Evaluating U at t = 0.06 i.e j = 2

at i = 1

$U_{1,3} = rU_{0,2} + (1-2r)U_{1,2} + rU_{2,2}$

$= 0.5(0) + (0)(0.0328) + 0.5(0.1152)$

$= 0.0576$

at i = 2

$U_{2,3} = rU_{1,2} + (1-2r)U_{2,2} + rU_{3,2}$

$= 0.5(0.0328) + 0(0.1152) + 0.5(0.3152)$

$= 0.1740$

at i = 3

$U_{3,3} = rU_{2,2} + (1-2r)U_{5,2} + rU_{4,2}$

$= 0.5(0.1152) + (0)(0.3152) + 0.5(0.6088)$

$= 0.3620$

at i = 4

$U_{4,3} = rU_{3,2} + (1-2r)U_{4,2} + rU_{5,2}$

$= 0.5(0.3152) + 0(0.6088) + 0.5(1)$

$= 0.6576$

$U_{0,2} = 0$

$U_{5,3} = 1$

Evaluating U at t = 0.08 i.e j = 3

at i = 1

$U_{1,4} = rU_{0,3} + (1-2r)U_{1,3} + rU_{2,3}$

$= 0.5(0) + 0(0.0570) + 0.5(0.1740)$

$= 0.087$

at i = 2

$U_{2,4} = rU_{1,3} + (1-2r)U_{2,3} + rU_{3,3}$

$= 0.5(0.0576) + 0(0.174) + 0.5(0.362)$

$= 0.2078$

at i = 3

$U_{3,4} = rU_{2,3} + (1-2r)U_{3,5} + rU_{4,3}$

$= 0.5(0.174) + 0(0.362) + 0.5(0.6576)$

$= 0.4158$

at i = 4

$U_{4,4} = rU_{3,3} + (1-2r)U_{4,3} + rU_{5,3}$

$= 0.5(0.362) + (0.6576) + 0.5(1)$

$= 0.681$

$U_{0,5} = 0$

$U_{5,5} = 1$

T \ x	0	0.2	0.4	0.6	0.8	1
0	0	0.0016	0.0256	0.1296	0.4096	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.1	0	0.1049	0.2514	0.4454	0.7079	1