

# Engineering Mathematics IV - ENG 382

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17/ENG 03/052

- Civil Engineering

## Assignment 6 - Solution

$$- \quad u_t - C u_{xx} = 0$$

$$\frac{du}{dt} - C \frac{d^2u}{dx^2} = 0$$

$$\frac{du}{dt} = C \frac{d^2u}{dx^2}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = C \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}$$

$$u_{i,j+1} - u_{i,j} = C \frac{\Delta t}{\Delta x^2} [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}] \times \frac{\Delta t}{\Delta x^2}$$

$$u[x, 0] = u^4 k.$$

$\Delta k = 0.2 \text{ m}$ ,  $\Delta t = 0.02 \text{ days}$   
for Initial Conditions

$$\text{At } x = 0 \rightarrow x^4 = 0$$

$$\text{At } x = 0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x = 0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x = 0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x = 0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x = 1 \rightarrow 1^4 = 1$$

$$\therefore u_{ij+1} = u_{ij} + r [u_{i+1j} - 2u_{ij} + u_{i-1j}]$$

$$u_{ij+1} = r u_{i-1j} + (1-2r)u_{ij} + r u_{i+1j} \rightarrow *$$

when  $i=1, j=0$

$$\therefore u_{1,0} = 0.5(u_{0,0}) + 0.5(u_{2,0})$$

$$= 0.5(0) + 0.5(0.0256)$$
$$= 0.0128$$

when  $i=2, j=0$

$$u_{2,0} = 0.5(u_{1,0}) + 0.5(u_{3,0})$$

$$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$= 0.0656$$

when  $i=3, j=0$

$$u_{3,0} = 0.5(u_{2,0}) + 0.5(u_{4,0})$$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$= 0.2176$$

when  $i=4, j=0$

$$u_{4,0} = 0.5(u_{3,0}) + 0.5(u_{5,0})$$

$$= 0.5(0.256) + 0.5(0.4096) = 0.5648$$

for  $j = 1$

$$\begin{aligned}\text{when } i = 1, \quad u_{1,2} &= 0.5(u_{0,1}) + 0.5(u_{2,1}) \\ &= 0.5 u_{0,1} + 0.5 u_{2,1} \\ &= 0 + 0.5(0.0656) = 0.0328\end{aligned}$$

$$\begin{aligned}\text{when } i = 2, \quad u_{2,2} &= 0.5(u_{1,1}) + 0.5(u_{3,1}) \\ &= 0.5(0.0128) + 0.5(0.2176) \\ &= 0.1152\end{aligned}$$

$$\begin{aligned}\text{when } i = 3, \quad u_{3,2} &= 0.5(u_{0,1}) + 0.5(u_{4,1}) \\ &= 0.5(0.0656) + 0.5(0.5648) \\ &= 0.3152\end{aligned}$$

$$\begin{aligned}\text{when } i = 4, \quad u_{4,2} &= 0.5(u_{2,1}) + 0.5(u_{5,1}) \\ &= 0.5(0.2176) + 0.5(1) \\ &= 0.6088\end{aligned}$$

for  $j = 2$

$$\begin{aligned}\text{when } i = 1, \quad u_{1,3} &= 0.5(u_{0,1}) + 0.5(u_{2,2}) \\ &= 0 + 0.5(0.1152) \\ &= 0.0576\end{aligned}$$

$$\text{when } i=2, u_{213} = 0.5(u_{112}) + 0.5(u_{312})$$

$$(1) \text{€} \cdot 0 + (2) \text{€} \cdot 0 = 0.5(0.0328) + 0.5(0.3152)$$

$$1 \text{€} = 0.174$$

$$\text{when } i=3, u_{313} = 0.5(u_{212}) + 0.5(u_{412})$$

$$(1) \text{€} \cdot 0 + (2) \text{€} \cdot 0 = 0.5(0.1152) + 0.5(0.6088)$$

$$(1) \text{€} \cdot 0 + (2) \text{€} \cdot 0 = 0.362$$

$$\text{when } i=4, u_{414} = 0.5(u_{312}) + 0.5(u_{512})$$

$$(1) \text{€} \cdot 0 + (2) \text{€} \cdot 0 = 0.5(0.3152) + 0.5(1)$$

$$(2) \text{€} \cdot 0 + (1) \text{€} \cdot 0 = 0.6576$$

$$1 \text{€} \cdot 0 =$$

for  $j=3$

$$\text{when } i=1, u_{114} = 0.5(u_{013}) + 0.5(u_{213})$$

$$(1) \text{€} \cdot 0 + (2) \text{€} \cdot 0 = 0.5(0) + 0.5(0.174)$$

$$1 \text{€} = 0.084$$

$$\text{when } i=2, u_{214} = 0.5(u_{113}) + 0.5(u_{313})$$

$$(1) \text{€} \cdot 0 + (2) \text{€} \cdot 0 = 0.5(0.0576) + 0.5(0.362)$$

$$1 \text{€} = 0.2098$$

$$\text{when } i=3, u_{314} = 0.5(u_{213}) + 0.5(u_{413})$$

$$= 0.5(0.174) + 0.5(0.6576) = 0.4158$$

$$\begin{aligned} \text{when } i=4, \quad u_{4,4} &= 0.5(u_{3,3}) + 0.5(u_{5,3}) \\ &= 0.5(0.362) + 0.5(1) \\ &= 0.681 \end{aligned}$$

for  $j=4$

$$\begin{aligned} \text{when } i=1, \quad u_{1,5} &= 0.5(u_{0,4}) + 0.5(u_{2,4}) \\ &= 0.5(0.098) = 0.1049 \end{aligned}$$

$$\begin{aligned} \text{when } i=2, \quad u_{2,5} &= 0.5(u_{1,4}) + 0.5(u_{3,4}) \\ &= 0.5(0.087) + 0.5(0.4158) \\ &= 0.2514 \end{aligned}$$

$$\begin{aligned} \text{when } i=3, \quad u_{3,5} &= 0.5(u_{2,4}) + 0.5(u_{4,4}) \\ &= 0.5(0.2098) + 0.5(0.681) \\ &= 0.4454 \end{aligned}$$

$$\begin{aligned} \text{when } i=4, \quad u_{4,5} &= 0.5(u_{3,4}) + 0.5(u_{5,4}) \\ &= 0.5(0.4158) + 0.5(1) \\ &= 0.7079 \end{aligned}$$

$\Delta t$	J Temp (K)						
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1
0.08	4	0	0.0870	0.2098	0.4158	0.6810	1
0.06	3	0	0.5760	0.1740	0.3620	0.6576	1
0.04	2	0	0.0328	0.1152	0.2152	0.6008	1
0.02	1	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0	0.0016	0.0256	0.1296	0.4096	1
$\Delta x$		0	0.2	0.4	0.6	0.8	1
	i	0	1	2	3	4	5

0.1	0	0.1049	0.2514	0.4454	0.7079	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0.0016	0.0256	0.1296	0.4096	1
	0	0.2	0.4	0.6	0.8	1

0.5

