

Mkpouto ubong obot

Civil Engineering

17/ENG03/015

ENG382 Assgn. 6

$$u_t - c u_{xx} = 0$$

$$\frac{du}{dt} - c \frac{d^2u}{dx^2} = 0$$

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$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = c \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}$$

$$u_{i,j+1} - u_{i,j} = c \frac{\Delta t}{\Delta x^2} [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}] \cdot r = \frac{\Delta t}{\Delta x^2}, l=1$$

$$u(x,0) = x^4$$

$$\Delta x = 0.2 \text{ m}, \Delta t = 0.02 \text{ day}$$

for the initial conditions

$$\text{At } x=0 \rightarrow x^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$\therefore u_{i,j+1} = u_{i,j} + r [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$$

$$u_{i,j+1} = r u_{i-1,j} + (1-2r) u_{i,j} + r u_{i+1,j} \quad *$$

when  $i=1, j=0$

$$u_{1,1} = 0.5 u_{0,0} + 0.5 u_{2,0}$$
$$= 0.5(0) + 0.5(0.0256)$$

$$u_{1,1} = 0.0128$$

when  $i=2, j=0$

$$u_{2,1} = 0.5(u_{1,0}) + 0.5(u_{3,0})$$
$$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$u_{2,1} = 0.0656$$

$$\text{When } i=3, j=0 = u_{3,1} = 0.5(u_{2,0}) + 0.5(u_{4,0}) \\ = 0.5(0.0256) + 0.5(0.4096)$$

$$u_{3,1} = 0.2176$$

$$\text{When } i=4, j=0 = u_{4,1} = 0.5(u_{3,0}) + 0.5(u_{5,0}) \\ = 0.5(0.0256) + 0.5(0.4096)$$

$$u_{4,1} = 0.5648$$

for  $j=1$

$$\text{When } i=1 \therefore u_{1,2} = 0.5(u_{0,1}) + 0.5(u_{2,1}) \\ = 0.5 u_{0,1} + 0.5 u_{2,1} = 0 + 0.5(0.0656)$$

$$u_{1,2} = 0.0328$$

$$\text{When } i=2 \therefore u_{2,2} = 0.5(u_{1,1}) + 0.5(u_{3,1}) \\ = 0.5(0.0128) + 0.5(0.2176)$$

$$u_{2,2} = 0.1152$$

$$\text{When } i=3 \quad u_{3,2} = 0.5(u_{0,1}) + 0.5(u_{4,1}) \\ = 0.5(0.0656) + 0.5(0.5648)$$

$$u_{3,2} = 0.3152$$

$$\text{When } i=4 \quad u_{4,2} = 0.5(u_{2,1}) + 0.5(u_{5,1}) \\ = 0.5(0.2176) + 0.5(1)$$

$$u_{4,2} = 0.6088$$

for  $J=2$

$$\text{When } i=1 \therefore u_{1,3} = 0.5(u_{0,2}) + 0.5(u_{2,2}) \\ = 0 + 0.5(0.1152) = 0.0576$$

$$\text{When } i=2 \quad u_{2,3} = 0.5(u_{1,2}) + 0.5(u_{3,2}) \\ = 0.5(0.0328) + 0.5(0.3152) = 0.174$$

$$\text{When } i=3 \quad u_{3,3} = 0.5(u_{2,2}) + 0.5(u_{4,2}) \\ = 0.5(0.1152) + 0.5(0.6088) = 0.362$$

$$\text{When } i=4 \quad u_{4,3} = 0.5(u_{3,2}) + 0.5(u_{5,2})$$

$$= 0.5(0.3152) + 0.5(1)$$

$$u_{4,3} = 0.6576$$

for  $J=3$

$$\text{When } i=1 \quad u_{1,4} = 0.5(u_{0,3}) + 0.5(u_{2,3})$$

$$= 0.5(0) + 0.5(0.174) = 0.084$$

$$\text{When } i=2 \quad u_{2,4} = 0.5(u_{1,3}) + 0.5(u_{3,3})$$

$$= 0.5(0.0576) + 0.5(0.362) = 0.2098$$

$$\text{When } i=3 \quad u_{3,4} = 0.5(u_{2,3}) + 0.5(u_{4,3})$$

$$= 0.5(0.174) + 0.5(0.6576) = 0.4158$$

$$\text{When } i=4 \quad u_{4,4} = 0.5(u_{3,3}) + 0.5(u_{5,3})$$

$$= 0.5(0.362) + 0.5(1)$$

$$= 0.681$$

for  $J=4$

$$\text{When } i=1 \quad u_{1,5} = 0.5(u_{0,4}) + 0.5(u_{2,4})$$

$$= 0.5(0.2098) = 0.1049$$

$$\text{When } i=2 \quad u_{2,5} = 0.5(u_{1,4}) + 0.5(u_{3,4})$$

$$= 0.5(0.087) + 0.5(0.4158) = 0.2514$$

$$\text{When } i=3 \quad u_{3,5} = 0.5(u_{2,4}) + 0.5(u_{4,4})$$

$$= 0.5(0.2098) + 0.5(0.681)$$

$$= 0.4454$$

$$\text{When } i=4 \quad u_{4,5} = 0.5(u_{3,4}) + 0.5(u_{5,4})$$

$$= 0.5(0.24158) + 0.5(1) = 0.7079$$

	J							
$\Delta t$	Temp (K)							
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1	
0.08	4	0	0.087	0.2098	0.4158	0.681	1	
0.06	3	0	0.0576	0.174	0.362	0.6576	1	
0.04	2	0	0.0328	0.1152	0.3152	0.6008	1	
0.02	1	0	0.0128	0.0656	0.2176	0.5648	1	
0	0	0	0.0016	0.028	0.1296	0.4096	1	
$\Delta x$		0	0.2	0.4	0.6	0.8	1	
	i	0	1	2	3	4	5	

