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17(ENG03)033 Civil Engineering

$$U_t - (U_{xx}) = 0$$

$$\frac{du}{dt} - c \frac{d^2 u}{dx^2} = 0$$

$$\frac{du}{dt} = c \frac{d^2 u}{dx^2}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = c \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}$$

$$u_{i,j+1} - u_{i,j} = \frac{(\Delta t [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}])}{\Delta x^2} \cdot r \cdot \frac{\Delta t}{\Delta x^2}, c=1$$

$$u[x_0] = 3c^4 k$$

$$\Delta x = 0.2m, \Delta t = 0.02$$

for initial Conditions

$$\text{At } x=0 \longrightarrow 0c^4 = 0$$

$$\text{At } x=0.2 \longrightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \longrightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \longrightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \longrightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \longrightarrow 1^4 = 1$$

$$\therefore u_{i,j+1} = u_{i,j} + r [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$$

$$U_{i,j+1} = \rho U_{i,j} + (1-2\rho)U_{i,j} + \tau U_{i+1,j}$$

When $i=1, j=0$

$$\begin{aligned} U_{0,1} &= 0.5 U_{0,0} + 0.5 U_{2,0} \\ &= 0.5(0) + 0.5(0.0256) \end{aligned}$$

$$U_{0,1} = 0.0128$$

When $i=2, j=0$

$$\begin{aligned} U_{2,1} &= 0.5(U_{1,0}) + 0.5(U_{0,0}) \\ &= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296) \end{aligned}$$

$$U_{2,1} = 0.0656$$

$$\begin{aligned} \text{When } i=3, j=0 - U_{3,1} &= 0.5(U_{2,0}) + 0.5(U_{1,0}) \\ &= 0.5(0.0256) + 0.5(0.4096) \end{aligned}$$

$$U_{3,1} = 0.2196$$

$$\begin{aligned} \text{When } i=4, j=0 - U_{4,1} &= 0.5(U_{3,0}) + 0.5(U_{2,0}) \\ &= 0.5(0.0256) + 0.5(0.4096) \end{aligned}$$

$$U_{4,1} = 0.5648$$

for $j=1$

$$\begin{aligned} \text{when } i=1 \quad \therefore U_{1,2} &= 0.5(U_0) + 0.5(U_{2,1}) \\ &= 0.5 U_{0,1} + 0.5 U_{2,1} = 0 + 0.5(0.0656) \end{aligned}$$

$$U_{1,2} = 0.0328$$

When $i=2$ $\therefore U_{2,2} = 0.5(U_{1,1}) + 0.5(U_{3,1})$
 $= 0.5(0.0128) + 0.5(0.2176)$
 $U_{2,2} = 0.1152$

When $i=3$ $\therefore U_{3,2} = 0.5(U_{2,1}) + 0.5(U_{4,1})$
 $= 0.5(0.0656) + 0.5(0.5648)$
 $U_{3,2} = 0.3152$

When $i=4$ $U_{4,2} = 0.5(U_{2,1}) + 0.5(U_{5,1})$
 $= 0.5(0.2176) + 0.5(1)$
 $U_{4,2} = 0.6088$

for $J=2$ *

when $i=1$ $\therefore U_{1,3} = 0.5(U_{0,1}) + 0.5(U_{2,2})$
 $= 0.5 + 0.5(0.1152) = 0.0576$

when $i=2$ $\therefore U_{2,3} = 0.5(U_{1,2}) + 0.5(U_{3,2})$
 $= 0.5(0.328) + 0.5(0.3152) = 0.174$

when $i=3$ $\therefore U_{3,3} = 0.5(U_{2,2}) + 0.5(U_{4,2})$
 $= 0.5(0.1152) + 0.5(0.6088) = 0.362$

when $i=4$ $\therefore U_{4,3} = 0.5(U_{3,2}) + 0.5(U_{5,2})$
 $= 0.5(0.3152) + 0.5(1) = 0.6576$

for $J=3$

$$\text{when } i = 1 \therefore U_{114} = 0.5(U_{013}) + 0.5(U_{213}) \\ = 0.5(0) + 0.5(0.174) = 0.084$$

$$\text{when } i = 2 \therefore U_{214} = 0.5(U_{113}) + 0.5(U_{313}) \\ = 0.5(0.0576) + 0.5(0.362) = 0.2098$$

$$\text{when } i = 3 \therefore U_{314} = 0.5(U_{213}) + 0.5(U_{413}) \\ = 0.5(0.174) + 0.5(0.6576) = 0.4158$$

$$\text{when } i = 4 \therefore U_{414} = 0.5(U_{313}) + 0.5(U_{513}) \\ 0.5(0.362) + 0.5(1) = 0.681$$

for $J=4$

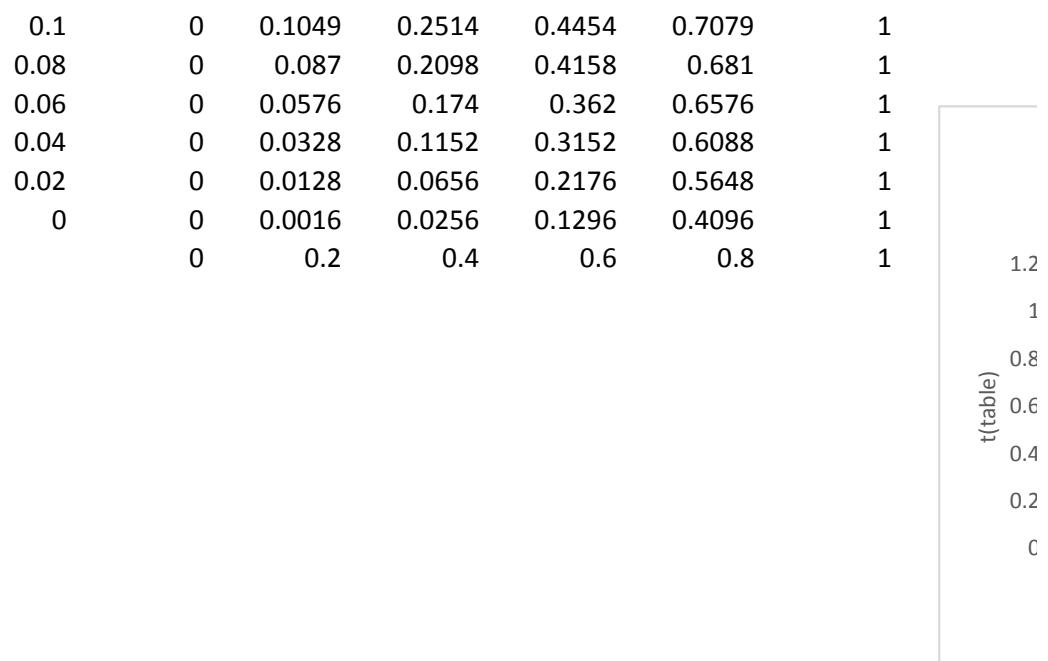
$$\text{when } i = 1 \therefore U_{115} = 0.5(U_{014}) + 0.5(U_{214}) \\ = 0.5(0.2098) + 0.5(0.1049) = 0.1573$$

$$\text{when } i = 2 \therefore U_{215} = 0.5(U_{114}) + 0.5(U_{314}) \\ = 0.5(0.087) + 0.5(0.4158) = 0.2514$$

$$\text{when } i = 3 \therefore U_{315} = 0.5(U_{214}) + 0.5(U_{414}) \\ = 0.5(0.2098) + 0.5(0.681) = 0.4454$$

$$\text{when } i = 4 \therefore U_{415} = 0.5(U_{314}) + 0.5(U_{514}) \\ = 0.5(0.4158) + 0.5(1) = 0.7079$$

Δt	J	Time (s)	0	0.1049	0.2514	0.4454	0.7079	1
0.1	5	0	0	0.087	0.2098	0.4158	0.681	1
0.08	4	0	0	0.0576	0.174	0.362	0.6576	1
0.06	3	0	0	0.0328	0.1152	0.3152	0.6008	1
0.04	2	0	0	0.0128	0.0656	0.2176	0.5648	1
0.02	1	0	0	0.0016	0.028	0.1296	0.4096	1
0	0	0	0	0.2	0.4	0.6	0.8	1
Δx	i	0	1	2	3	4	5	



0.5

3D REPRESENTATION OF TEMPERATURE CHANGE

