Mechanical Engineering
17/Eng06/106

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Use (U_{xx} for $0 \leq x \leq 1m$, $0 \leq t \leq 0.1$ day $\Delta x = 0.2m$

$U_t = cU_{xx}$

Initial condition

$U(x,0) = x^4 K = f(x)$

Boundary conditions

~~$U(x,0) = x^4$~~

$U(0,t) = 0 \leftarrow \cdots \rightarrow U(1,t) = 1K$

at $t = 0$;

$U_{0,0} = 0^2 K$ when $x = 0.4m$, $U_{2,0} = (0.4)^4$

$= 0.0256$

when $x = 0.8$, $U_{4,0} = (0.8)^4$

$= 0.4096$

when $x = 0.2m$

$U_{1,0} = (0.2)^4$

$= 0.0016$

when $x = 0.6m$

$U_{3,0} = (0.6)^4$

$= 0.1296$

when $x = 1$

$U_{5,0} = (1)^4$

$= 1$

① $U_t = cU_{xx} = 0$

$U_t = cU_{xx}$

$\frac{du}{dt} = c\frac{du}{dx}$ for $0.6 \leq x \leq 1$

$0 \leq t \leq 0.1$ day

Initial Condition

$U(x,0) = x^4 K$ $t = 0$ & $0 \leq x \leq 1$

K

BC

$U(0,t) = 0$

$U(1,t) = 1$

$0 \leq t \leq 0.1$ day

$$U_{i,j+1} - U_{ij} = \frac{C\Delta t}{\Delta x^2}\left[\frac{U_{i+1} - 2U_{ij} + U_{ij} + U_{i-1,j}}{\Delta x^2}\right]$$

$$U_{i,j+1} - U_{ij} = \frac{C\Delta t}{\Delta x^2}\left[U_{i+1,j} - 2U_{i,j} + U_{i-1,j}\right]$$

$$\frac{C\Delta t}{\Delta x^2} = \gamma$$

$$U_{ij+1} - U_{i,j} = r(U_{i+1,j} - 2U_{i,j} + U_{i-1,j})$$

$$U_{ij+1} = r(U_{i+1,j} - 2U_{i,j} + U_{i-1,j}) + U_{i,j}$$

$$U_{i,j+1} = r[U_{i+1,j} + (1-2r)U_{ij} + U_{i-1,j}]$$

$$r = (\Delta t/\Delta x^2) \quad 1 \times \frac{0.02}{(0.2)^2} = \frac{0.02}{0.04} = 0.5$$

From the equation

$$U_{i+1,j} = U_{i,j} + r[U_{i+1,j} - 2U_{i,j} + U_{i-1,j}]$$

at $j = 0$

$$U_{i,1} = U_{1,0} + r[U_{2,0} - 2U_{1,0} + U_{0,0}]$$

$$= 16\times10^{-3} + 0.5[0.0256 - 2(1.6\times10^{-3}) + 0]$$

$$U_{1,1} = 0.0178$$

$$i+1 = 2$$

$$U_{2,1} = U_{2,0} + r[U_{3,0} - 2U_{2,0} + U_{1,0}]$$

$$= 0.0256 + 0.5[0.1296 - 2[0.0256] + 1.6\times10^{-3}]$$

$$U_{2,1} = 0.0656$$

$$i+1 = 3$$

$$U_{3,1} = U_{3,0} + r[U_{4,0} - 2U_{3,0} + U_{2,0}]$$

$$= 0.1296 + 0.5[0.4096 - 2[0.1296] + 0.0256]$$

$$U_{3,1} = 0.2176$$

$$i+1 = 4$$

$U_{4,1} = U_{4,0} + r[U_{5,0} - 2U_{4,0} + U_{3,0}]$

$= 0.4096 + 0.5\,[1 - 2[0.4096 + 0.1296]$

$U_{4,1} = 0.5648$

At $J = 1$

$U_{i,2} = U_{i,1} + r[U_{i+1,j} - 2U_{i,1} + U_{i-1,1}]$

at $i = 1$

$U_{1,2} = U_{1,1} + 5\,[U_{2,1} - 2U_{1,1} + U_{0,1}]$

$= 0.0128 + 0.5\,[0.0656 - 2[0.028] + 0]$

$U_{1,2} = 0.0328$

at $i = 2$

$U_{2,2} = U_{2,1} + r[U_{3,1} - 2U_{2,1} + U_{1,1}]$

$= 0.0656 + 0.5[0.2176 - 2[0.0656] + 0.0128]$

$= 0.1152$

at $i = 3$

$U_{3,2} = U_{3,1} + r[U_{4,1} - 2U_{3,1} + U_{2,1}]$

$= 0.2176 + 0.5\,[0.5648 - 2(0.2176 + 0.0656)$

$U_{3,2} = 0.3152$

at $i = 4$

$U_{4,2} = U_{4,1} + r[U_{5,1} - 2U_{4,1} + U_{3,1}]$

$= 0.5648 + 0.5[1 - 2[0.5648) + (0.2176]$

$= 0.6088$

$\Rightarrow J = 2$

$U_{i,3} = U_{i,2} + r[U_{i+1,2} - 2U_{i,2} + U_{i-1,2}]$

at $i = 1$

$U_{1,3} = U_{1,2} + r[U_{2,2} - 2U_{1,2} + U_{0,2}]$

$= 0.0328 + 0.5[0.1152 - 2[0.0328] + 0]$

$= 0.0576$

at $i = 2$

$U_{2,3} = U_{2,2} + r[U_{3,2} - 2U_{2,2} + U_{1,2}]$

$= 0.1152 + 0.5[0.3152 - 2[0.1152] + 0.0328]$

$= 0.174$

At $i = 3$

$u_{3,3} = u_{3,2} + r[u_{4,2} - 2u_{3,2} + u_{2,2}]$

$= 0.3152 + 0.5[0.6088 - 2[0.3152] + 0.1157]$

$= 0.3620$

At $i = 4$

$u_{4,3} = u_{4,2} + r[u_{5,2} - 2u_{4,2} + u_{3,2}]$

$= 0.6088 + 0.5[1 - 2[0.6088] + 0.3152]$

$u_{4,3} = 0.6576$

$J = 3$

$u_{i,4} = u_{i,3} + r[u_{i+1,3} - 2u_{i,3} + u_{i-1,3}]$

At $i = 1$

$u_{1,4} = u_{1,3} + r[u_{2,3} - 2u_{1,3} + u_{0,3}]$

$= 0.0576 + 0.5[0.174 - 2[0.0576] + 0]$

$u_{1,4} = 0.087$

At $i = 2$

$u_{2,4} = u_{2,3} + r[u_{3,3} - 2u_{2,3} + u_{1,3}]$

$= 0.174 + 0.5[0.362 - 2[0.174] + 0.0576]$

$= 0.2098$

At $i = 3$

$u_{4,4} = u_{4,3} + r[u_{5,3} - 2u_{4,3} + u_{3,3}]$

$= 0.6576 + 0.5[1 - 2[0.6576] + 0.362]$

$= 0.681$

$J = 4$

$u_{1,5} = u_{1,4} + r[u_{i+1,4} - 2u_{i,4} + u_{i-1,4}]$

At $i = 1$

$= 0.087 + 0.5[0.2098 - 2[0.087) + 0]$

$u_{1,5} = 0.1049$

At i = 2

$u_{2,5} = u_{2,4} + r[u_{3,4} - 2u_{2,4} + u_{1,4}]$

$= 0.2098 + 0.5[0.4158 - 2[0.2098] + 0.097]$

$= 0.2514$

At i = 3

$u_{3,5} = u_{3,4} + 0.5[u_{4,4} - 2u_{3,4} + u_{2,4}]$

$= 0.4158 + 0.15[0.681 - 2[0.4158] + 0.2098]$

$= 0.4454$

At i = 4

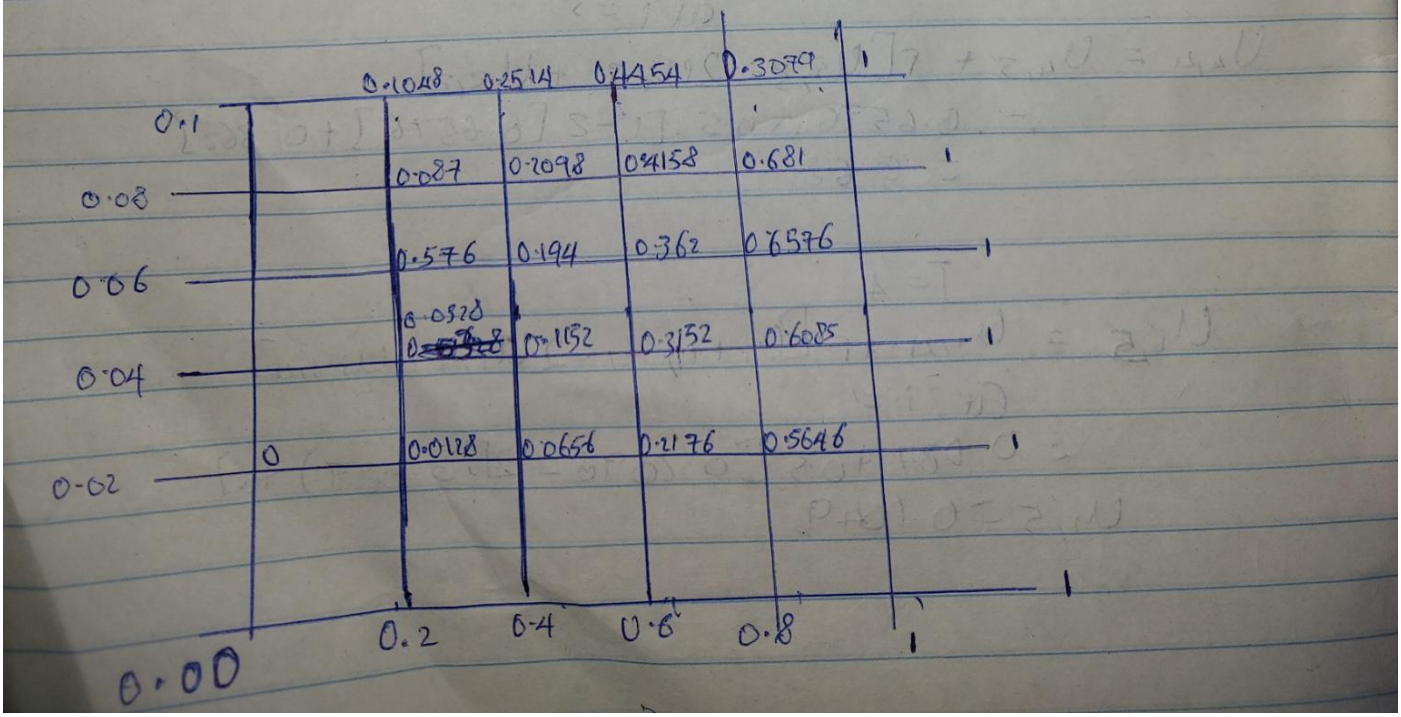
$u_{4,5} = u_{4,4} + r[u_{5,4} - 2u_{4,4} + u_{3,4}]$

$= 0.681 + 0.5[1 - 2[0.681] + 0.4158]$

$= 0.7077$

Note at $u_{0,5} = 0$ and $u_{5,5} = 1$

T/x	0	0.2	0.4	0.6	0.8	1
0	0	0.0016	0.0256	0.1296	0.4096	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0328	0.1152	0.3152	0.6048	1
0.06	0	0.0576	0.194	0.362	0.6576	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.1	0	0.1049	0.2314	0.4454	0.7079	1



Sheet1

	A	B	C	D	E	F	G	H	I	J	K	L
1		x=	0	0.2	0.4	0.6	0.8	1	Δt	Δx	r	
2	t	j/i	0	1	2	3	4	5	0.02	0.2	0.5	
3	0	0	0	0.0016	0.0256	0.1296	0.4096	1	0.02	0.2	0.5	
4	0.02	1	0	0.0128	0.0656	0.2176	0.5648	1	0.02	0.2	0.5	
5	0.04	2	0	0.0328	0.1152	0.3152	0.6088	1	0.02	0.2	0.5	
6	0.06	3	0	0.0576	0.174	0.362	0.6576	1	0.02	0.2	0.5	
7	0.08	4	0	0.087	0.2098	0.4158	0.681	1	0.02	0.2	0.5	
8	0.1	5	0	0.1049	0.2514	0.4454	0.7079	1	0.02	0.2	0.5	
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10	17/Eng06/106											
11	Mechanical Engineering											
12				300 level								
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