

17/ENG03/025

Idowu Olayinka

17/ENG03/025, Civil Engineering

$$\Delta t - C \Delta x = 0$$

$$\frac{\Delta x}{\Delta t} = C \frac{\Delta x^2}{\Delta t^2} = 0$$

$$\frac{\Delta x}{\Delta t} = C \frac{\Delta x^2}{\Delta t^2}$$

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta t} = C \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{\Delta x^2}$$

$$U_{i,j+1} - U_{i,j} = \frac{C \Delta t}{\Delta x^2} (U_{i,j+1} - 2U_{i,j} + U_{i,j-1})$$

$$U_{i,j+1} = U_{i,j} + \frac{C \Delta t}{\Delta x^2} (U_{i,j+1} - 2U_{i,j} + U_{i,j-1})$$

$$\Delta x = 0.2 \text{ m}, \Delta t = 0.02 \text{ day}$$

Boundary conditions

$$\Delta t \quad x=0 \rightarrow x \Delta t = 0$$

$$\Delta t \quad x=0.2 \rightarrow 0.2 \Delta t = 1.6 \times 10^{-3}$$

$$\Delta t \quad x=0.4 \rightarrow 0.4 \Delta t = 0.0256$$

$$\Delta t \quad x=0.6 \rightarrow 0.6 \Delta t = 0.1296$$

$$\Delta t \quad x=0.8 \rightarrow 0.8 \Delta t = 0.4096$$

$$\Delta t \quad x=1 \rightarrow 1 \Delta t = 1$$

$$\therefore U_{i,j+1} = U_{i,j} + \frac{C \Delta t}{\Delta x^2} (U_{i,j+1} - 2U_{i,j} + U_{i,j-1})$$

When $i=1, j=0$

$$U_{1,1} = 0.5U_{0,0} + 0.5U_{2,0}$$

$$= 0.5(0) + 0.5(0.0256)$$

$$U_{1,1} = 0.0128$$

When $i=2, j=0$

$$U_{2,1} = 0.5(U_{1,0}) + 0.5(U_{3,0})$$

$$= 0.5(1.67 \times 10^{-3}) + 0.5(0.1296)$$

$$U_{2,1} = 0.0656$$

$$= 0.5(0.0256) + 0.5(0.4096)$$

when $i=3, j=0, U_{3,1} = 0.2176$

when $i=4, j=0, U_{4,1} = 0.5(0.0256) + 0.5(0.4096)$
 $U_{4,1} = 0.5648$

For $j=1$

when $i=1, U_{1,2} = 0.5(U_{0,1}) + 0.5(U_{2,1})$
 $= 0.5U_{0,1} + 0.5U_{2,1} = 0 + 0.5(0.056)$
 $U_{1,2} = 0.0328$

when $i=2, U_{2,2} = 0.5(0.0128) + 0.5(0.2176)$
 $U_{2,2} = 0.1152$

when $i=3, U_{3,2} = 0.5(U_{0,1}) + 0.5(U_{4,1})$
 $= 0.5(0.0256) + 0.5(0.5648)$
 $U_{3,2} = 0.3152$

when $i=4, U_{4,2} = 0.5(U_{0,1}) + 0.5(U_{5,1})$
 $= 0.5(0.0256) + 0.5(1)$
 $U_{4,2} = 0.6088$

For $j=2$

when $i=1, U_{1,3} = 0 + 0.5(0.1152)$
 $= 0.0576$

$i=2, U_{2,3} = 0.5(0.0328) + 0.5(0.3152)$
 $= 0.174$

$i=3, U_{3,3} = 0.5(0.1152) + 0.5(0.6088)$
 $= 0.362$

$i=4, U_{4,3} = 0.5(0.3152) + 0.5(1)$
 $U_{4,3} = 0.6576$

For $j=3$

$i=1, U_{1,4} = 0.5(U_{0,3}) + 0.5(U_{2,3})$
 $= 0.5(0) + 0.5(0.174)$
 $= 0.084$

Ex 3

$$i=1, \quad U_{1,4} = 0.5(0) + 0.5(0.174) \\ = 0.087$$

$$i=2, \quad U_{2,4} = 0.2098$$

$$i=3, \quad U_{3,4} = 0.5(0.174) + 0.5(0.6576) = 0.4158$$

$$i=4 = 0.5(0.362) + 0.5(1) \\ U_{4,4} = 0.681$$

Ex 4

$$\text{When } i=1 \rightarrow U_{1,5} = 0.5(0) + 0.5(0.087) \\ U_{1,5} = 0.5(0.2098) = 0.1049$$

$$\text{When } i=2, \quad U_{2,5} = 0.5(U_{1,5}) + 0.5(U_{3,4}) \\ = 0.5(0.1049) + 0.5(0.4158) \\ = 0.25185$$

$$i=3 = \quad U_{3,5} = 0.5(0.2098) + 0.5(0.681) \\ = 0.4454$$

$$i=4, \quad 0.5(U_{3,5}) + 0.5(U_{5,4}) \\ = 0.5(0.4454) + 0.5(1) \\ = 0.7077$$

0.1	0	0.1049	0.2514	0.4454	0.7079	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0.0016	0.0256	0.1296	0.4096	1
	0	0.2	0.4	0.6	0.8	1

0.5

