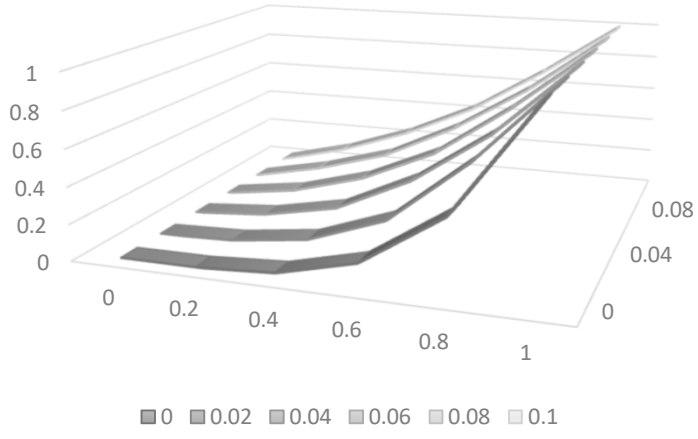


	0	0.2	0.4	0.6	0.8	1
0	0	0.0016	0.0256	0.1296	0.4096	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.1	0	0.1049	0.2514	0.4454	0.7079	1

Chart Title



UNIVERSITY OF SRI SRI SRI  
 17ENG051034  
 Mechatronics Engineering  
 ENG 382

Assignment

$u_x = (u_{xx} \text{ for } 0 \leq x \leq 1\text{m}, 0 \leq t \leq 0.1\text{day}; \Delta x = 0.2\text{m}$

$u_t = C u_{xx}$

Initial Condition:

$u(x, 0) = x^2 k = f(x)$

Boundary Condition

$u(0, t) = 0K \rightarrow u(1, t) = 1K$

At  $t=0$

$u_{0,0} = 0^2 k$

when  $x = 0.4\text{m}$

$u_{2,0} = (0.4)^2$   
 $= 0.16$

when  $x = 0.2\text{m}$

$u_{1,0} = (0.2)^2$   
 $= 0.04$

when  $x = 0.6\text{m}$

$u_{3,0} = (0.6)^2$   
 $= 0.36$

To find the temperature within the condition gradient

Using Euler method,

$u_{i,j+1} = r u_{i-1,j} + (1-2r) u_{i,j} + r u_{i+1,j}$

where  $r = \frac{C \Delta t}{\Delta x^2} = 1 \times \frac{0.02}{(0.2)^2} = 0.5$

Evaluating  $u$  at  $t = 0.02$  i.e.  $j = 0, i = 1, 2, 3, 4$

$u_{0,1} = 0(B.C)$

at  $i = 1$

$u_{1,1} = r u_{0,0} + (1-2r) u_{1,0} + r u_{2,0}$   
 $= 0.5(0) + (1-2(0.5)) \cdot 0.04 + 0.5(0.16)$   
 $= 0.08$

at  $i = 2$

$u_{2,1} = r u_{1,0} + (1-2r) u_{2,0} + r u_{3,0}$   
 $= 0.5(0.04) + (0)(0.16) + 0.5(0.36)$

$$= 0.0656$$

at  $i=3$

$$\begin{aligned} u_{3,i} &= r u_{2,0} + (1-2r) u_{2,0} + u_{4,0} \\ &= 0.5(0.0256) + (1-2(0.5))(0.1296) + 0.5(0.4096) \\ &= 0.2176 \end{aligned}$$

at  $i=4$

$$\begin{aligned} u_{4,i} &= r u_{3,0} + (1-2r) u_{4,0} + r u_{5,0} \\ &= 0.5(0.1296) + (1-2(0.5))(0.4096) + 0.5(0) \\ &= 0.5648 \end{aligned}$$

$$u_{5,i} = 1K$$

$$u_{1,1} = 0.0128 \quad u_{2,1} = 0.0656, \quad u_{3,1} = 0.2176, \quad u_{4,1} = 0.5648$$

Evaluating  $\mu$  at  $t=0.04$   $i=j=1$

at  $i=1$

$$\begin{aligned} u_{1,2} &= r u_{0,i} + (1-2r) u_{1,i} + r u_{2,i} \\ &= 0.5(0) + (0)(0.0128) + 0.5(0.0656) \\ &= 0.0328 \end{aligned}$$

at  $i=2$

$$\begin{aligned} u_{2,2} &= r u_{1,i} + (1-2r) u_{2,i} + r u_{3,i} \\ &= 0.5(0.0328) + 0(0.0656) + 0.5(0.2176) \\ &= 0.1152 \end{aligned}$$

at  $i=3$

$$\begin{aligned} u_{3,2} &= r u_{2,i} + (1-2r) u_{3,i} + r u_{4,i} \\ &= 0.5(0.0656) + 0(0.2176) + 0.5(0.5648) \\ &= 0.3152 \end{aligned}$$

at  $i=4$

$$\begin{aligned} u_{4,2} &= r u_{3,i} + (1-2r) u_{4,i} + r u_{5,i} \\ &= 0.5(0.2176) + 0(0.5648) + 0.5(0) \\ &= 0.6088 \end{aligned}$$

$$u_{0,2} = 0 \quad \text{and} \quad u_{5,2} = 1$$

Evaluating  $\mu$  at  $t=0.06$ ,  $i=j=2$

at  $i=1$

$$\begin{aligned} u_{1,3} &= r u_{0,2} + (1-2r) u_{1,2} + r u_{2,2} \\ &= 0.5(0) + (0)(0.0328) + 0.5(0.1152) \\ &= 0.0576 \end{aligned}$$

at  $i=2$

$$\begin{aligned} u_{2,3} &= r u_{1,2} + (1-2r) u_{2,2} + r u_{3,2} \\ &= 0.5(0.0576) + (0)(0.1152) + 0.5(0.3152) \\ &= 0.1740 \end{aligned}$$

at  $i = 3$

$$u_{3,3} = r u_{2,2} + (1-2r) u_{3,2} + r u_{4,2} \\ = 0.5(0.452) + 0(3.62) + 0.5(0.6088) \\ = 0.3620$$

at  $i = 4$

$$u_{4,3} = r u_{3,2} + (1-2r) u_{4,2} + r u_{5,2} \\ = 0.5(0.3152) + 0(0.6088) + 0.5(1) \\ = 0.6576$$

$$u_{0,3} = 0 \text{ and } u_{5,3} = 1$$

Evaluating  $u$  at  $t = 0.0812$ ,  $j = 3$

at  $i = 3$

$$u_{3,4} = r u_{0,3} + (1-2r) u_{3,3} + r u_{3,3} \\ = 0.5(0) + 0(0.0576) + 0.5(0.1740) \\ = 0.087$$

at  $i = 2$

$$u_{2,4} = r u_{1,3} + (1-2r) u_{2,3} + r u_{3,3} \\ = 0.5(0.0576) + 0(0.1740) + 0.5(0.362) \\ = 0.2098$$

at  $i = 3$

$$u_{3,4} = r u_{2,3} + (1-2r) u_{3,3} + r u_{4,3} \\ = ~~0.5(0) + 0(0.0576) + 0.5(0.1740)~~ \\ = ~~0.087~~ \\ = 0.5(0.1740) + 0(0.3620) + 0.5(0.6576) \\ = 0.4158$$

at  $i = 4$

$$u_{4,4} = r u_{3,3} + (1-2r) u_{4,3} + r u_{5,3} \\ = 0.5(0.3620) + 0(0.6576) + 0.5(1) \\ = 0.681$$

$$u_{0,4} = 0 \text{ and } u_{5,4} = 1$$

Evaluating  $u$  at  $t = 1$ , i.e.  $j = 4$

at  $i = 1$

$$u_{1,5} = r u_{0,4} + (1-2r) u_{1,4} + r u_{2,4} \\ = 0.5(0) + 0(0.087) + 0.5(0.2098) \\ = 0.1049$$

at  $i = 2$

$$u_{2,5} = r u_{1,4} + (1-2r) u_{2,4} + r u_{3,4} \\ = 0.5(0.087) + 0(0.2098) + 0.5(0.4158) \\ = 0.2514$$

at  $i = 3$

$$u_{3,5} = r u_{2,4} + (1-2r) u_{3,4} + r u_{4,4} \\ = 0.5(0.2098) + 0(0.4158) + 0.5(0.681) \\ = 0.4454$$

$at = 4$

$$u_{2,5} = r u_{2,4} + (1-r) u_{4,4} + F u_{5,4}$$
$$= 0.5(0.4158) + 0(0.681) + 0.5(1)$$
$$= 0.7079$$

<del><math>x</math></del>	0	0.2	0.4	0.6	0.8	1
0	0	0.0016	0.0256	0.1296	0.4096	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.06	0	0.0576	0.194	0.362	0.6576	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.1	0	0.1049	0.2514	0.4451	0.7079	1