

ADARA INUMBONG FERD

FI/ENGG02/025

ASSIGNMENT VI

$$U_x = U_{xx} = 0$$

$$\frac{du}{dt} - \frac{Cd^2u}{dx^2} = 0$$

$$\frac{du}{dt} = \frac{Cd^2u}{dx^2}$$

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta t} = \frac{(U_{i+1,j} - 2U_{i,j} + U_{i-1,j}))}{\Delta x^2} \approx 0$$

$$U_{i,j+1} - U_{i,j} = \left(\frac{\Delta t}{\Delta x^2} (U_{i+1,j} - 2U_{i,j} + U_{i-1,j}) \right) \Gamma$$
$$\Gamma = \frac{\Delta t}{\Delta x^2} C; C = 1, \Gamma = 0.5, \Gamma = 0.5$$

$$U(x, 0) = x^4 k$$

$$\Delta x = 0.2 \text{ m}, \Delta t = 0.02 \text{ day}$$

for initial conditions

$$\text{At } x=0 \rightarrow x^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$U_{i,j+1} = U_{i,j} + \Gamma (U_{i+1,j} - 2U_{i,j} + U_{i-1,j})$$
$$U_{i,j+1} = \Gamma (U_{i-1,j} + (1-2\Gamma)U_{i,j} + U_{i+1,j})$$

when $i=1, j=0$

$$U_{1,1} = 0.5 U_{0,0} + 0.5 U_{2,0}$$

$$0.5(0) + 0.5(0.0256) = 0.0128$$

$$U_{1,1} = 0.0128$$

when $i=2, j=0$

$$U_{2,1} = 0.5 (U_{1,0}) + 0.5 (U_{3,0})$$

$$0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$U_{2,1} = 0.0656$$

when $i=3, j=0$

$$U_{3,1} = 0.5(U_{2,0}) + 0.5(U_{4,0})$$
$$= 0.5(0.0256) + 0.5(0.4096)$$

$$U_{3,1} = 0.2176$$

when $i=4, j=0$

$$U_{4,1} = 0.5(U_{3,0}) + 0.5(U_{5,0})$$
$$= 0.5(0.0256) + 0.5(0.6656)$$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$U_{4,1} = 0.5648$$

for $j=1$

when $i=1$

$$U_{1,2} = 0.5(U_{0,1}) + 0.5(U_{2,1})$$
$$= 0.5(0) + 0.5(0.0656)$$

$$U_{1,2} = 0.0328$$

when $i=2$

$$U_{2,2} = 0.5(U_{1,1}) + 0.5(U_{3,1})$$

$$= 0.5(0.0328) + 0.5(0.2176)$$

$$U_{2,2} = 0.1152$$

when $i=3$

$$U_{3,2} = 0.5(U_{2,1}) + 0.5(U_{4,1})$$

$$= 0.5(0.0656) + 0.5(0.5648)$$

$$U_{3,2} = 0.3152$$

when $i=4$

$$U_{4,2} = 0.5(U_{3,1}) + 0.5(U_{5,1})$$

$$= 0.5(0.2176) + 0.5(1)$$

$$U_{4,2} = 0.6088$$

For $T=2$

when $i=1$

$$\begin{aligned} \therefore U_{1,3} &= 0.5(U_{0,1}) + 0.5(U_{2,2}) \\ &= 0 + 0.5(0.152) = 0.076 \end{aligned}$$

when $i=2$

$$\begin{aligned} U_{2,3} &= 0.5(U_{1,2}) + 0.5(U_{3,2}) \\ &= 0.5(0.0328) + 0.5(0.3152) = 0.174 \end{aligned}$$

when $i=3$

$$\begin{aligned} U_{3,3} &= 0.5(U_{2,2}) + 0.5(U_{4,2}) \\ &= 0.5(0.152) + 0.5(0.6088) = \\ &= 0.382 \end{aligned}$$

when $i=4$

$$\begin{aligned} U_{4,3} &= 0.5(U_{3,2}) + 0.5(S,2) \\ &= 0.5(0.3152) + 0.5(1) = \\ &= 0.6576 \end{aligned}$$

For $T=3$

when $i=1$

$$\begin{aligned} U_{1,4} &= 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ &= 0.5(0) + 0.5(0.174) = 0.087 \end{aligned}$$

when $i=2$

$$\begin{aligned} U_{2,4} &= 0.5(U_{1,3}) + 0.5(U_{3,3}) \\ &= 0.5(0) + 0.5(0.382) = \\ &= 0.191 \end{aligned}$$

when $i=3$

$$\begin{aligned} U_{3,4} &= 0.5(U_{2,3}) + 0.5(U_{4,3}) \\ &= 0.5(0.174) + 0.5(0.6576) = \\ &= 0.4158 \end{aligned}$$

For $J=2$

$$\text{when } i=1 \quad U_{1,4} = 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ = 0.5(0) + 0.5(0.174) \\ = 0.087$$

when $i=2$

$$U_{2,4} = 0.5(U_{1,3}) + 0.5(U_{3,3}) \\ = 0.5(0.087) + 0.5(0.342) \\ = 0.2098$$

when $i=3$

$$U_{3,4} = 0.5(U_{2,2}) + 0.5(U_{4,2}) \\ = 0.5(0.174) + 0.5(0.6576) \\ = 0.4158$$

when $i=4$

$$U_{4,4} = 0.5(U_{3,3}) + 0.5(U_{5,3}) \\ = 0.5(0.362) + 0.5(1) \\ = 0.681$$

For $J=4$

when $i=1$

$$U_{1,5} = 0.5(U_{0,4}) + 0.5(U_{2,4}) \\ = 0.5(0.2098) + 0.5(0.1649) \\ = 0.18735$$

when $i=2$

$$U_{2,5} = 0.5(U_{1,4}) + 0.5(U_{3,4}) \\ = 0.5(0.087) + 0.5(0.4158) \\ = 0.2514$$

when $i=3$

$$U_{3,5} = 0.5(U_{2,4}) + 0.5(U_{4,4}) \\ = 0.5(0.2098) + 0.5(0.681) \\ = 0.4454$$

when $i=4$

$$U_{4,5} = 0.5(U_{3,4}) + 0.5(U_{5,4})$$

$$= 0.5(0.14158) + 0.5(1)$$

$$= 0.7079$$

Δt	$J/\text{Temp}(K)$			
0.1	5	0	0.1049	0.2514
0.08	4	0	0.087	0.2098
0.06	3	0	0.2576	0.174
0.04	2	0	0.0328	0.1152
0.02	1	0	0.0128	0.0656
0	0	0	0.0016	0.028
Δx		0	0.2	0.4
	L	0	1	2

0.4454	0.7079	1
0.4158	0.681	1
0.362	0.6576	1
0.3152	0.6008	1
0.2176	0.5648	1
0.1296	0.4098	1
0.6	0.8	1
3	4	5

