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17/ENG05/013

MECHATRONIC ENGINEERING

$$U_t - CU_{xx} = 0$$

$$\frac{du}{dt} - C \frac{d^2u}{dx^2} = 0$$

$$\frac{du}{dt} = C \frac{d^2u}{dx^2}$$

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta t} = C \frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{\Delta x^2}$$

$$U_{i,j+1} - U_{i,j} = \frac{C \Delta t}{\Delta x^2} (U_{i+1,j} - 2U_{i,j} + U_{i-1,j}) \quad \Gamma = \frac{C \Delta t}{\Delta x^2}; C=1$$

$$U(x,0) = x^4$$

$$\Delta x = 0.2 \text{ m}, \Delta t = 0.02 \text{ s}$$

for initial conditions

$$\text{At } x=0 \rightarrow x^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$\therefore U_{i,j+1} = U_{i,j} + r [U_{i+1,j} - 2U_{i,j} + U_{i-1,j}]$$

$$U_{i,j+1} = \Gamma U_{i-1,j} + (1-2\Gamma)U_{i,j} + \Gamma U_{i+1,j} \quad (*)$$

when $i=1, j=0$

$$U_{1,1} = 0.5 U_{0,0} + 0.5 U_{2,0}$$

$$U_{1,1} = 0.5(0) + 0.5(0.0256)$$

$$U_{1,1} = 0.0128$$

when $i=2, j=0$

$$U_{2,1} = 0.5(U_{1,0}) + 0.5(U_{3,0})$$

$$U_{2,1} = 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$U_{2,1} = 0.0656$$

when $i=3, j=0$

$$U_{3,1} = 0.5(U_{2,0}) + 0.5(U_{4,0})$$

$$U_{3,1} = 0.5(0.0256) + 0.5(0.4096)$$

$$U_{3,1} = 0.2176$$

when $i=4, j=0$

$$U_{4,1} = 0.5(U_{3,0}) + 0.5(U_{5,0})$$

$$U_{4,1} = 0.5(0.0256) + 0.5(0.4096)$$

$$U_{4,1} = 0.5648$$

when $j=1$

when $i=2$

$$\therefore U_{1,2} = 0.5(U_{0,1}) + 0.5(U_{2,1})$$

$$= 0.5U_{0,1} + 0.5U_{2,1} = 0 + 0.5(0.0656)$$

$$= 0.0328$$

when $i=2$

$$\therefore U_{2,2} = 0.5(U_{1,1}) + 0.5(U_{3,1})$$

$$= 0.5(0.0128) + 0.5(0.2176)$$

$$= 0.1152$$

when $i=3$

$$\therefore U_{3,2} = 0.5(U_{0,1}) + 0.5(U_{4,1})$$

$$= 0.5(0.0656) + 0.5(0.5648)$$

$$= 0.3152$$

when $i=4$

$$\therefore U_{4,2} = 0.5(U_{2,1}) + 0.5(U_{5,1})$$

$$= 0.5(0.2176) + 0.5(1)$$

$$= 0.6088$$

when $j=2$

when $i=1$

$$\therefore U_{1,3} = 0.5(U_{0,1}) + 0.5(U_{2,2})$$

$$= 0 + 0.5(0.1152) = 0.0576$$

when $i=2$

$$\therefore U_{2,3} = 0.5(U_{1,2}) + 0.5(U_{3,2})$$

$$= 0.5(0.0328) + 0.5(0.3152)$$

$$= 0.174$$

when $i=3$

$$\therefore U_{3,3} = 0.5(U_{2,2}) + 0.5(U_{4,2})$$

$$= 0.5(0.1152) + 0.5(0.6088)$$

$$= 0.362$$

when $i=4$

$$\therefore U_{4,3} = 0.5(U_{3,1}) + 0.5(U_{3,2})$$

$$= 0.5(0.3152) + 0.5(1)$$

$$= 0.6576$$

When J=3

when $i=1$

$$\begin{aligned} \therefore U_{1,4} &= 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ &= 0.5(0) + 0.5(0.134) = 0.067 \end{aligned}$$

when $i=2$

$$\begin{aligned} \therefore U_{2,4} &= 0.5(U_{1,3}) + 0.5(U_{3,3}) \\ &= 0.5(0.0576) + 0.5(0.362) \\ &= 0.2098 \end{aligned}$$

when $i=3$

$$\begin{aligned} \therefore U_{3,4} &= 0.5(U_{2,3}) + 0.5(U_{4,3}) \\ &= 0.5(0.124) + 0.5(0.6576) \\ &= 0.4158 \end{aligned}$$

when $i=4$

$$\begin{aligned} \therefore U_{4,4} &= 0.5(U_{3,3}) + 0.5(U_{5,3}) \\ &= 0.5(0.362) + 0.5(1) \\ &= 0.681 \end{aligned}$$

For J=4

when $i=1$

$$\begin{aligned} \therefore U_{1,5} &= 0.5(U_{0,4}) + 0.5(U_{2,4}) \\ &= 0.5(0.2098) \\ &= 0.1049 \end{aligned}$$

when $i=2$

$$\begin{aligned} \therefore U_{2,5} &= 0.5(U_{1,4}) + 0.5(U_{3,4}) \\ &= 0.5(0.087) + 0.5(0.4158) \\ &= 0.2514 \end{aligned}$$

when $i=3$

$$\begin{aligned} \therefore U_{3,5} &= 0.5(U_{2,4}) + 0.5(U_{4,4}) \\ &= 0.5(0.2098) + 0.5(0.681) \\ &= 0.4454 \end{aligned}$$

when $i=4$

$$\begin{aligned} \therefore U_{4,5} &= 0.5(U_{3,4}) + 0.5(U_{5,4}) \\ &= 0.5(0.4158) + 0.5(1) \\ &= 0.7079 \end{aligned}$$

J	Temp (K)					
5	0	0.1049	0.2514	0.4454	0.7079	1
4	0	0.0876	0.2078	0.4158	0.6810	1
3	0	0.0576	0.1740	0.3620	0.6576	1
2	0	0.0328	0.1152	0.3152	0.6008	1
1	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0.0016	0.0280	0.1296	0.4096	1
	0	0.0000	0.0000	0.6000	0.8000	1
6	0	1	2	3	4	5