

Name: Shokunbi Emmanuel

Dept: Mechanical Engineering

MAT no: 17ENGG01075

Course code: ENG382

$$U_t - CU_{xx} = 0 \quad \text{for } 0 \leq x \leq 1\text{m}, 0 \leq t \leq 0.1\text{day}$$

Initial condition,  $U(x,0) = x^4/K, 0 \leq x \leq 1\text{m}$

$$U(0,t) = 0K, U(1,t) = 1K \quad \text{for } 0 \leq t \leq 0.1\text{day}$$

$$\text{(a) } x = 0\text{m}, x = 1\text{m} \quad \Delta x = 0.2\text{m}, \Delta t = 0.02\text{day}$$

$$C = 1 \frac{\text{m}^2}{\text{day}}$$

$$U_t - CU_{xx} = 0$$

$$\therefore U_t - U_{xx} = 0, C = 1$$

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} = 0$$

Initial condition,

$$U(x,0) = x^4/K$$

$$\text{@ } x = 0.2, U_{2,0} = (0.2)^4 = 1.6 \times 10^{-3}K$$

$$\text{@ } x = 0.4, U_{4,0} = (0.4)^4 = 0.0256K$$

$$\text{@ } x = 0.6, U_{6,0} = (0.6)^4 = 0.1296K$$

$$\text{@ } x = 0.8, U_{8,0} = (0.8)^4 = 0.4096K$$

from initial cond.,  $U_{0,0} = 0K, U_{5,0} = 1K$

$$r = \frac{\Delta t}{(\Delta x)^2} = \frac{0.02}{(0.2)^2} = \frac{0.02}{0.04}$$

$$r = \frac{1}{2} = 0.5$$

$$1 - 2r = 1 - 2(0.5)$$

$$= 1 - 1 = 0$$

$$\therefore (1 - 2r) = 0$$

When  $x=1, t=0$

$$U_{1,1} = r U_{0,0} + (1 - 2r)U_{1,0} + r U_{2,0} = 0 + 0 + 0.5(0.0256) = 0.0128$$

$$x=2 \quad U_{2,1} = r U_{1,0} + (1 - 2r)U_{2,0} + r U_{3,0} = 0.5(1.6 \times 10^{-3}) + 0 + 0.5(0.1296) = 0.0656$$

$$x=3 \quad U_{3,1} = r U_{2,0} + (1 - 2r)U_{3,0} + r U_{4,0} = 0.5(0.0256) + 0 + 0.5(0.4096) = 0.2176$$

$$x=4 \quad U_{4,1} = r U_{3,0} + (1 - 2r)U_{4,0} + r U_{5,0} = 0.5(0.1296) + 0 + 0.5(1) = 0.5648$$

When  $x=0, t=0.02$ ,  $U_{0,1} = 0K; U_{5,1} = 1K$

When  $x=1, t=0.02$

$$U_{1,2} = r U_{0,1} + (1 - 2r)U_{1,1} + r U_{2,1} = 0.5(0) + 0 + 0.5(0.0656) = 0.0328K$$

$$x=2 \quad U_{2,2} = r U_{1,1} + (1 - 2r)U_{2,1} + r U_{3,1} = 0.5(0.0128) + 0 + 0.5(0.2176) = 0.1152K$$

$$x=3 \quad U_{3,2} = r U_{2,1} + (1 - 2r)U_{3,1} + r U_{4,1} = 0.5(0.0656) + 0 + 0.5(0.5648) = 0.3152K$$

$$x=4 \quad U_{4,2} = r U_{3,1} + (1 - 2r)U_{4,1} + r U_{5,1} = 0.5(0.2176) + 0 + 0.5(1) = 0.6088K$$

$U_{0,2} = 0K, U_{5,2} = 1K$

$$x=1, t=2$$

$$U_{1,3} = rU_{0,2} + (1-2r)U_{1,2} + rU_{2,2} = 0(0.5) + 0 + 0.5(0.1152) = 0.0576k$$

$$x=2$$

$$U_{2,3} = rU_{1,2} + (1-2r)U_{2,2} + rU_{3,2} = 0.5(0.0328) + 0 + 0.5(0.3152) = 0.174k$$

$$x=3$$

$$U_{3,3} = rU_{2,2} + (1-2r)U_{3,2} + rU_{4,2} = 0.5(0.1152) + 0 + 0.5(0.6088) = 0.362k$$

$$x=4$$

$$U_{4,3} = rU_{3,2} + (1-2r)U_{4,2} + rU_{5,2} = 0.5(0.3152) + 0 + 0.5(1) = 0.6576k$$

$$U_{0,3} = 0k, U_{5,3} = 1k$$

$$x=1, t=3$$

$$U_{1,4} = rU_{0,3} + (1-2r)U_{1,3} + rU_{2,3} = 0.5(0) + 0 + 0.5(0.174) = 0.087k$$

$$x=2$$

$$U_{2,4} = rU_{1,3} + (1-2r)U_{2,3} + rU_{3,3} = 0.5(0.0576) + 0 + 0.5(0.362) = 0.2098k$$

$$x=3$$

$$U_{3,4} = rU_{2,3} + (1-2r)U_{3,3} + rU_{4,3} = 0.5(0.174) + 0 + 0.5(0.6576) = 0.4158k$$

$$x=4$$

$$U_{4,4} = rU_{3,3} + (1-2r)U_{4,3} + rU_{5,3} = 0.5(0.362) + 0 + 0.5(1) = 0.681k$$

$$U_{0,4} = 0k, U_{5,4} = 1k$$

$$x=1, t=4$$

$$U_{1,5} = rU_{0,4} + (1-2r)U_{1,4} + rU_{2,4} = 0.5(0) + 0 + 0.5(0.2098) = 0.1049k$$

$$x=2$$

$$U_{2,5} = rU_{1,4} + (1-2r)U_{2,4} + rU_{3,4} = 0.5(0.087) + 0 + 0.5(0.4158) = 0.2514k$$

$$x=3$$

$$U_{3,5} = rU_{2,4} + (1-2r)U_{3,4} + rU_{4,4} = 0.5(0.2098) + 0 + 0.5(0.681) = 0.4454k$$

$$x=4$$

$$U_{4,5} = rU_{3,4} + (1-2r)U_{4,4} + rU_{5,4} = 0.5(0.4158) + 0 + 0.5(1) = 0.7079k$$

$$U_{0,5} = 0k, U_{5,5} = 1k$$

$\Delta t (k)$

Step No	Grid No	0	1	2	3	4	5		
1-0	0	0	0.1049	0.2514	0.4454	0.7079	1	0	
0-08	1	0	0.087	0.2098	0.4158	0.681	1	1	
0-06	2	0	0.0576	0.174	0.362	0.6576	1	2	
0-04	3	0	0.0328	0.1152	0.3152	0.6088	1	3	
0-02	4	0	0.0128	0.0656	0.2176	0.5648	1	4	
0	5	0	0.0016	0.0288	0.1296	0.4096	1	5	
		0	1	2	3	4	5		
		0	0.2	0.4	0.6	0.8	1.0		

$\Delta x (m)$

