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17/ENG01/010
CHEMICAL ENGINEERING

Answer

Using explicit forward difference method

$U_t = cU_{xx}$ for $0 \le x \le 1m$, $0 \le t \le 0.1$ day, $\Delta x = 0.2m$

$\therefore U_t = cU_{xx}$

Initial Condition

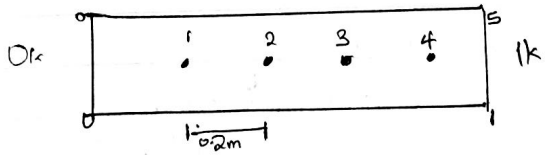
$U(x,0) = x^4 \; k = f(x)$

Boundary Condition

$U(0,t) = 0k \longrightarrow U(1,t) = 1k$

Graphically

$i=0, \; I=0 \; \therefore t=0$



Since it is at boundary condition, we use the function of x

At $t=0$

$U_{2,0} = 0k$

when $x = 0.4m$

$U_{2,0} = (0.4)^4$

$= 0.0256$

when $x = 0.8$

$U_{4,0} = (0.8)^4$

$= 0.4096$

when $x = 0.2$

$U_{1,0} = (0.2)^4$

$= 0.0016$

when $x = 0.6m$

$U_{3,0} = (0.6)^4$

$= 0.1296$

when $x = 1$

$U_{5,0} = (1)^4$

$= 1$

To find the temperature within the condition gradient, we use the discretized forward Euler method.

$$U_{i,j+1} = r U_{i-1,j} + (1-2r) U_{i,j} + r U_{i+1,j}$$

where

$$r = C\frac{\Delta t}{\Delta x^2} = 1 \times \frac{0.02}{(0.2)^2} = 0.5$$

Evaluating U at $t = 0.02$ i.e $j = 0$, $i = 1, 2, 3, 4$

$U_{0,1} = 0$ (BC)

at $i = 1$

$U_{1,1} = r U_{0,0} + (1-2r) U_0 + r U_{20}$

$= 0.5(0) + (1-2(0.5))0.0016 + 0.5(0.1296)$

at $i = 2$ $= 0 + 0 + 0.1296$

$U_{2,1} = r U_{1,0} + (1-2r) U_{2,0} + r U_{3,0} = 0.0128$

at $i = 2$, $U_{2,1} = r U_{1,0} + (1-2r) U_{2,0} + r U_{3,0}$

$= 0.5(0.0016) + 0(0.0256) + 0.5(0.126)$

$= 0.06526$

at $i = 3$

$U_{3,1} = U_{2,0} + (1-2r) U_{3,0} + r U_{4,0}$

$= 0.5(0.0256) + (1-2)(0.5))0.1296 + 0.5(0.4096)$

$= 0.2176$

at $i = 4$

$U_{4,1} = r U_{3,0} + (1-2r)(U_{4,0} + r U_{5,0}$

$= 0.5(0.1896) + (1-2(0.5))(0.4096)$

$+ 0.5(1)$

$= 0.5648$

$\therefore U_{1,1}, U_{2,1}, U_{3,1}, U_{4,1} =$

$U_{1,1} = 0.0128$

$U_{2,1} = 0.0656$

$U_{3,1} = 0.2176$

$U_{4,1} = 0.5648$

Evaluating U at $t = 0.01$ i.e $j = 1$

at $i = 1$

$U_{1,2} = r U_{2,1} + (1-2r) U_{1,1} + r U_{2,1}$

$= 0.5(0) + (0)(0.0128) + 0.5(0.06526)$

$= 0.0328$

at $i = 2$

$U_{2,2} = r U_{1,1} + (1-2r) U_{2,1} + r U_{3,1}$

$= 0.5(0.0128) + 0(0.0656) + 0.5(0.2176)$

$= 0.11552$

at $i = 3$

$U_{3,2} = r U_{2,1} + (1-2r) U_{3,1} + r U_{4,1}$

$= 0.5(0.0656) + 0(0.2176) +$

$0.5(0.5648)$

$= 0.3152$

at i = 4

$u_{4,2} = r(u_{3,1} + (1-2r)u_{4,1} + u_{5,1}$

$= 0.5(0.2176) + 0(0.5648) + 0.5(1)$

$= 0.6088$

NB $u_{0,2} = 0$ & $u_{5,2} = 1$ (due to Boundary condition)

Evaluating u at $t = 0.06$ i.e $j = 2$

at ~~$i = 1$~~ $i = 1$

$u_{1,3} = r(u_{0,2} + (1-2r)u_{1,2} + ru_{2,2}$

$= 0.5(0) + 0(0.3248) + 0.5(0.1152)$

$= 0.0576$

at $i = 2$

$u_{2,3} = r(u_{1,2} + (1-2r)u_{2,2} + ru_{3,2}$

$= 0.5(0.0328) + 0(0.1152) + 0.5$

(0.3152)

$= 0.1740$

at $i = 3$

$u_{3,3} = r(u_{2,2} + (1-2r)u_{3,2} + ru_{4,2}$

$= 0.5(0.1152) + (0)(0.3152) + 0.5$

(0.6088)

$= 0.3620$

at $i = 4$

$u_{4,3} = ru_{3,2} + (1-2r)u_{4,2} + ru_{5,2}$

$= 0.5(0.3152) + 0(0.6088) + 0.5(1)$

$= 0.6576$

NB = $u_{0,3} = 0$ & $u_{5,3} = 1$

Evaluating u at $t = 0.08$ i.e $j = 3$

Noted before : $u_{0,4} = 0$ & $u_{5,4} = 1$

at $i = 1$

$u_{1,4} = r(u_{0,3}) + (1-2r)u_{1,3} + ru_{2,3}$

$= 0.5(0) + 0(0.0576) + 0.5$

(0.1740)

$= 0.087$

at $i = 2$

$u_{2,4}$

at $= r(u_{1,3} + (1-2r)(u_{2,3}) + ru_{3,3}$

$= 0.5(0.0576) + 0.(0.1740) + 0.5$

(0.362)

$= 0.2098$

at $i=3$

$U_{3,4} = r U_{2,3} + (1-2r) U_{3,3} + r U_{4,3}$

$= 0.5(0.174) + 0(0.362) + 0.5$
(0.6576)

$= 0.4158$

at $i=4$

$U_{4,4} = r U_{3,3} + (1-2r) U_{4,3} + r U_{3,3}$

$= 0.5(0.362) + 0(0.6576) + 0.5(1)$

$= 0.681$

Evaluating U at $t=1$, $j=4$

at $i=1$

$U_{1,5} = r U_{0,4} + (1-2r) U_{1,4} + r U_{2,4}$

$= 0.5(0) + 0(0.087) + 0.5$
(0.2098)

$= 0.1049$

at $i=3$

$U_{3,5} = r U_{2,4} + (1-2r) U_{3,4} + r U_{4,4}$

$= 0.5(0.2098) + 0(0.4158) + 0.5$
(0.681)

$= 0.4454$

at $i=2$

$U_{2,5} = r U_{1,4} + (1-2r) U_{2,4} + r U_{3,4}$

$= 0.5(0.087) + 0(0.2098) + 0.5$
(0.4158)

$= 0.0435 + 0 + 0.2079$

$= 0.2514$

at $i=4$

$U_{4,5} = r U_{3,4} + (1-2r) U_{4,4} + r U_{5,4}$

$= 0.5(0.4158) + 0(0.681) + 0.5$
(1)

$= 0.7079$

N/B: $U_{0,5} = 0$ & $U_{5,5} = 1$

t \ x	0	0.2	0.4	0.6	0.8	1
0	0	0.0016	0.0256	0.1296	0.4096	1
0.2	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.06	0	0.0576	0.194	0.362	0.6576	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.1	0	0.1049	0.2514	0.4454	0.7079	1

TABULAR ~~FORM~~ FORM

1.b

	0	0.2	0.4	0.6	0.8	1
0	0	0.0016	0.0256	0.1296	0.4096	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.1	0	0.1049	0.2514	0.4454	0.7079	1