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Ue: (U_{xx} for $0 \leq x \leq 1m$, $0 \leq t \leq 0.1$ day $\Delta x = 0.2m$)

$$U_t = C U_{xx}$$

initial condition

$$U(x, 0) = \text{set } k = f(x)$$

Boundary conditions

$$U(0, t) = 0 \quad U(1, t) = 1k$$

at $t = 0$

$$U_{0,0} = 0^2 k \quad \text{when } x = 0.4, \quad U_{2,0} = (0.4)^4 = 0.0256$$

$$\text{when } x = 0.8, \quad U_{4,0} = (0.8)^4 = 0.4096$$

when $x = 0.2m$

$$U_{1,0} = (0.2)^4 = 0.0016$$

when $x = 0.6m$

$$U_{3,0} = (0.6)^4 = 0.1296$$

when $x = 1$

$$U_{5,0} = 1^4 = 1$$

1) $U_t = (U_{xx} = 0$

$$U_t = C U_{xx}$$

$$\frac{dy}{dt} = C \frac{dy}{dx} \quad \text{for } 0 \leq x \leq 1$$

$$0 \leq t \leq 0.1 \text{ day}$$

initial condition

$$U(x, 0) = x^4 k \quad t = 0 \quad \& \quad 0 \leq x \leq 1$$

Bc

$$U(0, t) = 0 \text{ K}$$

$$U(L, t) = K$$

$$0 \leq t \leq 0.1 \text{ day}$$

$$U_{i,j+1} - U_{i,j} = \frac{\Delta t}{\Delta x^2} \left[\frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{\Delta x^2} \right]$$

$$U_{i,j+1} - U_{i,j} = \frac{\Delta t}{\Delta x^2} [U_{i+1,j} - 2U_{i,j} + U_{i-1,j}]$$

$$\frac{\Delta t}{\Delta x^2} = 0$$

$$U_{i,j+1} - U_{i,j} = [U_{i+1,j} - 2U_{i,j} + U_{i-1,j}]$$

$$U_{i,j+1} - r(U_{i+1,j} - 2U_{i,j} + U_{i-1,j}) + U_{i,j}$$

$$U_{j,j+1} = [U_{i+1,j} + (1-r)U_{i,j} + U_{i-1,j}]$$

$$r = \frac{(\Delta t / \Delta x^2) \cdot 1 \times 0.02}{(0.2)^2} = \frac{0.02}{0.04} = 0.5$$

from the equation

$$U_{i,j+1} = U_{i,j} + r[U_{i+1,j} - 2U_{i,j} + U_{i-1,j}]$$

$$\text{at } j=0$$

$$U_{i,1} = U_{i,0} + r[U_{i+1,0} - 2U_{i,0} + U_{i-1,0}]$$

$$= 1.6 \times 10^{-3} + 0.5 [0.0250 - 2(1.6 \times 10^{-3}) + 0]$$

$$U_{1,1} = 0.0128$$

$$U_{2,1} = 2$$

$$U_{2,1} = U_{2,0} + r[U_{3,0} - 2U_{2,0} + U_{1,0}]$$

$$= 0.0250 + 0.5 [0.1296 - 2 [0.0256] + 1.6 \times 10^{-3}]$$

$$u_{2,1} = 0.0656$$

$$u_{i+1} = 3$$

$$\begin{aligned} u_{3,1} &= u_{3,0} + \tau [u_{4,0} - 2u_{3,0} + u_{2,0}] \\ &= 0.1296 + 0.5 [0.4096 - 2 [0.1296] + 0.0256] \end{aligned}$$

$$u_{3,1} = 0.2176$$

$$u_{i+1} = 4$$

$$\begin{aligned} u_{4,1} &= u_{4,0} + \tau [u_{5,0} - 2u_{4,0} + u_{3,0}] \\ &= 0.4096 + 0.5 [1 - 2 [0.4096] + 0.1296] \end{aligned}$$

$$u_{4,1} = 0.5944$$

$$\text{at } j = 1$$

$$u_{i,j+1} = u_{i,j} + \tau [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$$

$$\text{at } i = 1$$

$$\begin{aligned} u_{1,2} &= u_{1,1} + \tau [u_{2,1} - 2u_{1,1} + u_{0,1}] \\ &= 0.0123 + 0.5 [0.0656 - 2 [0.0123] + 0] \end{aligned}$$

$$u_{1,2} = 0.0328$$

$$\text{at } i = 2$$

$$\begin{aligned} u_{2,2} &= u_{2,1} + \tau [u_{3,1} - 2u_{2,1} + u_{1,1}] \\ &= 0.0656 + 0.5 [0.2176 - 2 [0.0656] + 0.0123] \\ &= 0.1152 \end{aligned}$$

$$\text{at } i = 3$$

$$\begin{aligned} u_{3,2} &= u_{3,1} + \tau [u_{4,1} - 2u_{3,1} + u_{2,1}] \\ &= 0.2176 + 0.5 [0.5944 - 2 [0.2176] + 0.0656] \end{aligned}$$

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$$u_{(3,2)} = 0.3152$$

at $i = 4$

$$\begin{aligned} u_{4,2} &= u_{4,1} + r [u_{5,1} - 2u_{4,1} + u_{3,1}] \\ &= 0.5648 + 0.5 [1 - 2(0.5648) + (0.2176)] \\ &= 0.6088 \\ &\Rightarrow \delta = 2 \end{aligned}$$

$$u_{i,3} = u_{i,2} + r [u_{i+1,2} - 2u_{i,2} + u_{i-1,2}]$$

at $i = 1$

$$\begin{aligned} u_{1,3} &= u_{1,2} + r [u_{2,2} - 2u_{1,2} + u_{0,2}] \\ &= 0.0378 + 0.5 [0.1152 - 2[0.0318] + 0] \\ &= 0.0576 \end{aligned}$$

at $i = 2$

$$\begin{aligned} u_{2,3} &= u_{2,2} + r [u_{3,2} - 2u_{2,2} + u_{1,2}] \\ &= 0.1152 + 0.5 [0.3152 - 2[0.1152] + 0.0378] \\ &= 0.174 \end{aligned}$$

at $i = 3$

$$\begin{aligned} u_{3,3} &= u_{3,2} + r [u_{4,2} - 2u_{3,2} + u_{2,2}] \\ &= 0.3152 + 0.5 [0.6088 - 2[0.3152] + 0.1152] \\ &= 0.3620 \end{aligned}$$

at $i = 4$

$$\begin{aligned} u_{4,3} &= u_{4,2} + r [u_{5,2} - 2u_{4,2} + u_{3,2}] \\ &= 0.6088 + 0.5 [1 - 2[0.6088] + 0.3152] \\ u_{4,3} &= 0.6576 \end{aligned}$$

$$j = 3$$

$$U_{1,4} = U_{1,3} + r [U_{1+1,3} - 2U_{1,3} + U_{1-1,3}]$$

at $l = 1$

$$U_{1,4} = U_{1,3} + r [U_{2,3} - 2U_{1,3} + U_{0,3}]$$

$$= 0.0576 + 0.5(0.174) - 2[0.0576] + 0$$

$$U_{1,4} = 0.097$$

at $i = 2$

$$U_{2,4} = U_{2,3} + r [U_{3,3} - 2U_{2,3} + U_{1,3}]$$

$$= 0.174 + 0.5[0.362 - 2(0.174) + 0.0576]$$

$$= 0.2099$$

at $i = 3$

$$U_{4,4} = U_{4,3} + r [U_{5,3} - 2U_{4,3} + U_{3,3}]$$

$$= 0.6576 + 0.5 [1 - 2(0.6576) + 0.362]$$

$$= 0.681$$

$$j = 4$$

$$U_{1,5} = U_{1,4} + r [U_{1+1,4} - 2U_{1,4} + U_{1-1,4}]$$

at $l = 1$

$$= 0.097 + 0.5 [0.7099 - 2(0.097) + 0]$$

$$U_{1,5} = 0.1049$$

$l + 1 = 2$

$$U_{2,5} = U_{2,4} + r [U_{3,4} - 2U_{2,4} + U_{1,4}]$$

$$= 0.2099 + 0.5 [0.4155 - 2(0.2099) + 0.097]$$

$$= 0.2514$$

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at $i = 3$

$$U_{3,5} = U_{3,4} + 0.5 (U_{4,4} - 2U_{3,4} + U_{2,4})$$

$$= 0.4158 + 0.5 [0.681 - 2[0.4158]] + 0.209$$

$$= 0.4454$$

at $i = 4$

$$U_{4,5} = U_{4,4} + 0.5 (U_{5,4} - 2U_{4,4} + U_{3,4})$$

$$= 0.681 + 0.5 [1 - 2[0.681] + 0.4156]$$

$$= 0.7079$$

Note at $U_{0,5} = 0$ and $U_{5,5} = 1$

T/x	0	0.2	0.4	0.6	0.8	1
0	0	0.0016	0.0256	0.1296	0.4096	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0320	0.1152	0.3152	0.6088	1
0.06	0	0.0576	0.194	0.362	0.6576	1
0.08	0	0.087	0.2098	0.4158	0.691	1
0.1	0	0.1049	0.2314	0.4454	0.7079	1

0.1	0.1049	0.2314	0.4454	0.7079	1
0.08	0.087	0.2098	0.4158	0.691	1
0.06	0.0576	0.194	0.362	0.6576	1
0.04	0.0320	0.1152	0.3152	0.6088	1
0.2	0.0128	0.0656	0.2176	0.5648	1

0.2 0.4 0.6 0.8 1