

Aravind Energy Bridge
 17th Floor
 Mechatronics Engineering

$$U_0 - (U_{n,n} = 0$$

$$\frac{\delta U}{\delta t} - c \frac{\delta^2 U}{\delta x^2} = 0$$

$$\frac{\delta U}{\delta t} = c \frac{\delta^2 U}{\delta x^2}$$

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta t} = c \frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{\Delta x^2}$$

$$U_{i,j+1} - U_{i,j} = \frac{c \Delta t}{\Delta x^2} (U_{i+1,j} - 2U_{i,j} + U_{i-1,j}) \quad \left[= \frac{\Delta t}{\Delta x^2} (c=1) \right]$$

$$U(x,0) = x^4 k$$

$$\Delta x = 0.2 \text{ m} \quad \Delta t = 0.02 \text{ sec}$$

For initial conditions

$$\text{At } x=0 \rightarrow u^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$U_{i,j+1} = U_{i,j} + r [U_{i+1,j} - 2U_{i,j} + U_{i-1,j}]$$

$$U_{i,j+1} = r U_{i-1,j} + (1-2r) U_{i,j} + r U_{i+1,j}$$

When $i=1, j=0$

$$U_{1,1} = 0.5 U_{0,0} + 0.5 U_{2,0}$$

$$= 0.5(0) + 0.5(0.0256)$$

$$U_{1,1} = 0.0128$$

When $i=2, j=0$

$$U_{2,1} = 0.5 (U_{1,0}) + 0.5 (U_{3,0})$$

$$= 0.5 (1.6 \times 10^{-3}) + 0.5 (0.1296)$$

$$U_{2,1} = 0.0656$$

when $\bar{i} = 3, j = 0 \Rightarrow U_{3,1} = 0.5(U_{2,0}) + 0.5(U_{4,0})$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$U_{3,1} = 0.2176$$

when $\bar{i} = 4, j = 0 \Rightarrow U_{4,1} = 0.5(U_{3,0}) + 0.5(U_{5,0})$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$U_{4,1} = 0.5648$$

For $j = 1$

when $\bar{i} = 1 \Rightarrow U_{1,2} = 0.5(0.1) + 0.5(U_{2,1})$

$$= 0.5U_{0,1} + 0.5U_{2,1} = 0 + 0.5(0.0656)$$

$$U_{1,2} = 0.0328$$

when $\bar{i} = 2 \Rightarrow U_{2,2} = 0.5(U_{1,1}) + 0.5(U_{3,1})$

$$= 0.5(0.0128) + 0.5(0.2176)$$

$$U_{2,2} = 0.1152$$

when $\bar{i} = 3 \Rightarrow U_{3,2} = 0.5(U_{2,1}) + 0.5(U_{4,1})$

$$= 0.5(0.0656) + 0.5(0.5648)$$

$$U_{3,2} = 0.3152$$

when $\bar{i} = 4 \Rightarrow U_{4,2} = 0.5(U_{3,1}) + 0.5(U_{5,1})$

$$= 0.5(0.2176) + 0.5(1)$$

$$U_{4,2} = 0.6088$$

$\bar{j} = 2$

when $\bar{i} = 1 \Rightarrow U_{1,3} = 0.5(U_{0,1}) + 0.5(U_{2,2})$

$$= 0 + 0.5(0.1152) = 0.0576$$

when $\bar{i} = 2 \Rightarrow U_{2,3} = 0.5(1,3) + 0.5(U_{3,2})$

$$= 0.5(0.0576) + 0.5(0.3152) = 0.2098$$

When $\bar{i} = 3$ $U_{3,4} = 0.5(U_{2,3}) + 0.5(U_{4,3})$
 $= 0.5(0.774) + 0.5(0.6576) = 0.4158$

When $\bar{i} = 4$ $U_{4,4} = 0.5(U_{3,3}) + 0.5(U_{5,3})$
 $= 0.5(0.362) + 0.5(1)$
 $= 0.681$

For $j = 4$

When $\bar{i} = 1$ $U_{1,5} = 0.5(U_{0,4}) + 0.5(U_{2,4})$
 $= 0.5(0.2098) = 0.1049$

When $\bar{i} = 2$ $U_{2,5} = 0.5(U_{1,4}) + 0.5(U_{3,4})$
 $= 0.5(0.087) + 0.5(0.4158) = 0.2514$

When $\bar{i} = 3$ $U_{3,5} = 0.5(U_{2,4}) + 0.5(U_{4,4})$
 $= 0.5(0.2098) + 0.5(0.681)$
 $= 0.4454$

When $\bar{i} = 4$ $U_{4,5} = 0.5(U_{3,4}) + 0.5(U_{5,4})$
 $= 0.5(0.4158) + 0.5(1)$
 $= 0.7079$

Δt	j temp (K)						
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1
0.08	4	0	0.087	0.2098	0.4158	0.681	1
0.06	3	0	0.0576	0.174	0.362	0.6576	1
0.04	2	0	0.0328	0.1152	0.3152	0.6008	1
0.02	1	0	0.0128	0.0654	0.2176	0.5698	1
0	0	0	0.0016	0.028	0.1246	0.4098	1
0.2		0	0.2	0.4	0.6	0.8	1
	i	0	1	2	3	4	5