

$$\begin{aligned} \text{when } i=6 \quad U_{6,6} &= 0.5(U_{6,6}) + 0.5(U_{6,5}) \\ &= 0.5(0.42) + 0.5(0.7) \\ &= 0.56 \end{aligned}$$

also for  $i=5$

$$\begin{aligned} \text{when } i=5 \quad U_{5,5} &= 0.5(U_{5,5}) + 0.5(U_{5,4}) \\ &= 0.5(0.29) + 0.5(0.7) \\ &= 0.49 \end{aligned}$$

$$\begin{aligned} \text{when } i=2 \quad U_{2,2} &= 0.5(U_{2,2}) + 0.5(U_{2,1}) \\ &= 0.5(0.17) + 0.5(0.6158) = 0.3929 \end{aligned}$$

$$\begin{aligned} \text{when } i=3 \quad U_{3,3} &= 0.5(U_{3,3}) + 0.5(U_{3,2}) \\ &= 0.5(0.2098) + 0.5(0.691) \\ &= 0.4504 \end{aligned}$$

$$\begin{aligned} \text{when } i=4 \quad U_{4,4} &= 0.5(U_{4,4}) + 0.5(U_{4,3}) \\ &= 0.5(0.4654) + 0.5(0.7) \\ &= 0.5827 \end{aligned}$$

$i \backslash j$	0	1	2	3	4	5
0.1	0	0	0.169	0.27%	0.42%	0.57%
0.01	0	0	0.067	0.2098	0.49	0.49
0.06	0	0	0.203	0.4	0.53	0.69%
0.04	0	0	0.438	0.652	0.562	0.69%
0.02	0	0	0.627	0.616	0.573	0.644
0	0	0	0.684	0.627	0.47%	0.60%
0.2	0	0	0.2	0.6	0.6	0.9
i	0	1	2	3	4	5

$$= 0.5(0) + 0.5(0.0256)$$

$$U_{11} = 0.0128$$

$$\text{when } i=2, j=0$$

$$U_{21} = 0.5(U_{11}) + 0.5(U_{10})$$

$$= 0.5(0.0128) + 0.5(0.0096)$$

$$U_{21} = 0.0112$$

$$\text{when } i=3, j=0$$

$$U_{31} = 0.5(U_{21}) + 0.5(U_{20})$$

$$= 0.5(0.0112) + 0.5(0.0064)$$

$$U_{31} = 0.0088$$

$$\text{when } i=4, j=0$$

$$U_{41} = 0.5(U_{31}) + 0.5(U_{30})$$

$$= 0.5(0.0088) + 0.5(0.0048)$$

$$U_{41} = 0.0068$$

for j=1

$$\text{when } i=1, j=1$$

$$U_{12} = 0.5(U_{11}) + 0.5(U_{01})$$

$$= 0.5(0.0128) + 0.5(0.0056)$$

$$U_{12} = 0.0092$$

$$\text{when } i=2, j=1$$

$$U_{22} = 0.5(U_{12}) + 0.5(U_{21})$$

$$= 0.5(0.0092) + 0.5(0.0112)$$

$$U_{22} = 0.0102$$

$$\text{when } i=3, j=1$$

$$U_{32} = 0.5(U_{22}) + 0.5(U_{31})$$

$$= 0.5(0.0102) + 0.5(0.0088)$$

$$U_{32} = 0.0095$$

$$\text{when } i=4, j=1$$

$$U_{42} = 0.5(U_{32}) + 0.5(U_{41})$$

$$= 0.5(0.0095) + 0.5(0.0068)$$

$$U_{42} = 0.00815$$

for T=2

$$\text{when } i=1$$

$$U_{11} = 0.5(U_{10}) + 0.5(U_{01})$$

$$= 0.5(0) + 0.5(0.0152) = 0.0076$$

$$\text{when } i=2$$

$$U_{21} = 0.5(U_{11}) + 0.5(U_{20})$$

$$= 0.5(0.0076) + 0.5(0.0072) = 0.0074$$

$$\text{when } i=3$$

$$U_{31} = 0.5(U_{21}) + 0.5(U_{30})$$

for T=2

$$\text{when } i=1 \quad U_{1,1} = 0.5(U_{1,1}) + 0.5(U_{1,2})$$

$$0.5(0.5) + 0.5(0.152) = 0.2574$$

$$\text{when } i=2 \quad U_{2,1} = 0.5(U_{2,1}) + 0.5(U_{2,2})$$

$$0.5(0.0395) + 0.5(0.207) = 0.1174$$

$$\text{when } i=3 \quad U_{3,1} = 0.5(U_{3,1}) + 0.5(U_{3,2})$$

$$0.5(0.005) + 0.5(0.099) = 0.042$$

$$\text{when } i=6 \quad U_{6,1} = 0.5(U_{6,1}) + 0.5(0)$$

$$0.5(0.442) + 0.5(0)$$

$$U_{6,1} = 0.221$$

for T=3

$$\text{when } i=1 \quad U_{1,1} = 0.5(U_{1,1}) + 0.5(U_{1,2})$$

$$0.5(0) + 0.5(0.04) = 0.02$$

$$\text{when } i=2 \quad U_{2,1} = 0.5(0) + 0.5(U_{2,1})$$

$$0.5(0.0576) + 0.5(0.367) = 0.2123$$

$$\text{when } i=3 \quad U_{3,1} = 0.5(U_{3,1}) + 0.5(U_{3,2})$$

$$0.5(0.076) + 0.5(0.099) = 0.0875$$

$$\text{when } i=6 \quad U_{6,1} = 0.5(U_{6,1}) + 0.5(U_{6,2})$$

$$0.5(0.442) + 0.5(0)$$

$$= 0.221$$

for T=4

$$\text{when } i=1 \quad U_{1,1} = 0.5(U_{1,1}) + 0.5(U_{1,2})$$

$$0.5(0.2098) + 0.5(0.069)$$

$$\text{when } i=2 \quad U_{2,1} = 0.5(U_{2,1}) + 0.5(U_{2,2})$$

$$0.5(0.0977) + 0.5(0.0119) = 0.0548$$

$$\text{when } i=3 \quad U_{3,1} = 0.5(U_{3,1}) + 0.5(U_{3,2})$$

$$0.5(0.2098) + 0.5(0.019)$$

$$= 0.1144$$

$$\text{when } i=6 \quad U_{6,1} = 0.5(U_{6,1}) + 0.5(U_{6,2})$$

$$0.5(0.06158) + 0.5(0)$$

## Assignment 6.

$$U_t = C U_{xx} + 0$$

$$\frac{du}{dt} = C \frac{d^2 u}{dx^2} = 0$$

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~~$$\frac{du}{dt} = C \frac{d^2 u}{dx^2} = 0$$~~

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta t} = C \frac{(U_{i,j} - 2U_{i,j} + U_{i-1,j}))}{\Delta x^2}$$

$$U_{i,j+1} - U_{i,j} = \frac{C \Delta t}{\Delta x^2} (U_{i,j} - 2U_{i,j} + U_{i-1,j}) \quad r = \frac{C \Delta t}{\Delta x^2}, \quad C = 1$$

$$U(x, 0) = x^2 k$$

$$\Delta x = 0.2m, \quad \Delta t = 0.02 \text{ day}$$

for initial condition

$$\text{At } x = 0 \rightarrow x^2 = 0$$

$$\text{At } x = 0.2 \rightarrow 0.2^2 = 16 \times 10^{-2}$$

$$\text{At } x = 0.4 \rightarrow 0.4^2 = 0.16$$

$$\text{At } x = 0.6 \rightarrow 0.6^2 = 0.36$$

$$\text{At } x = 0.8 \rightarrow 0.8^2 = 0.64$$

$$\text{At } x = 1 \rightarrow 1^2 = 1$$

$$\therefore U_{i,j+1} = U_{i,j} + r(U_{i,j} - 2U_{i,j} + U_{i-1,j})$$

$$U_{i,j+1} = r(U_{i,j} + (1-2r)U_{i,j} + rU_{i-1,j}) \quad x$$

when  $i=1, j=0$ 

$$U_{1,1} = 0.5(U_{1,0}) + 0.5(U_{0,0})$$

$$= 0.5(0) + 0.5(0.0256)$$

$$U_{1,1} = 0.0128$$

when  $i=2, j=0$ 

$$U_{2,1} = 0.5(U_{2,0}) + 0.5(U_{1,0})$$

$$0.5(16 \times 10^{-2}) + 0.5(0.16)$$