

Assignment

$$u_t - Cu_{xx} = 0$$

for $0 \leq x \leq 1\text{m}$, $0 \leq t \leq 0.1$ day

initial condition: $u(x, 0) = x^4 k$

for $0 \leq x \leq 1\text{m}$

Boundary condition: $u(0, t) = 0 k$

$u(1, t) = 1 k$, for $0 \leq t \leq 0.1$ day

if $\Delta x = 0.2$, $\Delta t = 0.02$ days

$$c = \frac{1\text{m}^2}{\text{day}}$$

Solution

We are applying boundary function which implies that use the function of x at $t = 0$

$$u_{0,0} = 0^2 k = 0$$

when $x = 0.4$

$$u_{2,0} = (0.4)^4 = 0.0256$$

when $x = 0.8$

$$u_{4,0} = (0.8)^4 = 0.4096$$

when $x = 0.2$

$$u_{1,0} = (0.2)^4$$

$$u_{1,0} = 0.0016$$

when $x = 0.6$

$$u_{3,0} = (0.6)^4 = 0.1296$$

when $x = 1.0$

$$u_{5,0} = (1.0)^4 = 1$$

Recall that $u_t - Cu_{xx} = 0$

$$\therefore u_t = Cu_{xx}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = c \frac{d^2 u}{dx^2}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = c \left[\frac{(u_{i+1,j}) - (2u_{i,j}) + (u_{i-1,j}))}{\Delta x^2} \right]$$

$$u_{i,j+1} - u_{i,j} = \frac{c \Delta t}{\Delta x^2} (u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) - (1)$$

$$\text{Let } r = \frac{c \Delta t}{\Delta x^2}$$

$$r = \frac{1 \cdot 0.02}{(0.2)^2} = 0.5$$

From equ (1)

$$i = 1 \quad j = 0$$

$$u_{i,j+1} = u_{i,j} + 0.5(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

$$u_{1,1} = u_{1,0} + 0.5(u_{2,0} - 2u_{1,0} + u_{0,0})$$
$$= 0.0016 + 0.5(0.0256 - 2(0.0016) + 0)$$

$$u_{1,1} = 0.0128$$

$$i=2, j=0$$

$$u_{i,j+1} = u_{i,j} + r(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

$$u_{2,1} = u_{2,0} + r(u_{3,0} - 2u_{2,0} + u_{1,0})$$
$$= 0.0256 + 0.5(0.1296 - 2(0.0256) + 0.0016)$$

$$\text{Recall that } = 0.0656$$

$$\text{when } i=3, j=0$$

$$u_{i,j+1} = u_{i,j} + 0.5(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

$$u_{3,1} = u_{3,0} + 0.5(u_{4,0} - 2u_{3,0} + u_{2,0})$$
$$= 0.1296 + 0.5(0.4096 - 2(0.1296) + 0.0256)$$
$$= 0.2176$$

$$\text{when } i=4, j=0$$

$$u_{i,j+1} = u_{i,j} + 0.5(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

$$u_{4,1} = u_{4,0} + 0.5(u_{5,0} - 2u_{4,0} + u_{3,0})$$
$$= 0.4096 + 0.5(1 - 2(0.4096) + 0.1296)$$
$$= 0.5648$$

$$\text{at } j=1$$

$$\text{when } i=1, j=1$$

$$u_{i,j+1} = u_{i,j} + 0.5(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

$$u_{1,2} = u_{1,1} + 0.5(u_{2,1} - 2u_{1,1} + u_{0,1})$$
$$= 0.0128 + 0.5(0.0656 - 2(0.0128) + 0)$$

$$u_{1,2} = 0.0328$$

$$\text{when } i=2$$

$$u_{i,j+1} = u_{i,j} + 0.5(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

$$= u_{2,1} + 0.5(u_{3,1} - 2u_{2,1} + u_{1,1})$$
$$= 0.0656 + 0.5(0.2176 - 2(0.0656) + 0.0128)$$

$$u_{2,2} = 0.1152$$

$$\text{when } i=3$$

$$u_{i,j+1} = u_{i,j} + 0.5(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

$$u_{3,2} = u_{3,1} + 0.5(u_{4,1} - 2u_{3,1} + u_{2,1})$$
$$= 0.2176 + 0.5(0.5648 - 2(0.2176) + 0.0656)$$

$$u_{3,2} = 0.3152$$

when $i = 4$

$$u_{i,j+1} = u_{i,j} + 0.5(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

$$u_{4,2} = u_{4,1} + 0.5(u_{5,1} - 2u_{4,1} + u_{3,1})$$
$$= 0.5648 + 0.5(u_{5,1} - 2u_{4,1} + u_{3,1})$$
$$= 0.6088$$

at $j = 2$

$$= u_{1,2} + 0.5(u_{2,2} - 2u_{1,2} + u_{0,2})$$
$$= 0.0328 + 0.5(0.1152 - 2(0.0328) + 0)$$

$$u_{1,3} = 0.0576$$

when $i = 2$

$$u_{2,3} = u_{2,2} + 0.5(u_{3,2} - 2u_{2,2} + u_{1,2})$$
$$= 0.1152 + 0.5(0.3152 - 2(0.1152) + 0.0328)$$

$$u_{2,3} = 0.174$$

when $i = 3$

$$u_{3,3} = u_{3,2} + 0.5(u_{4,2} - 2u_{3,2} + u_{2,2})$$
$$= 0.3152 + 0.5(0.6088 - 2(0.3152) + 0.1152)$$
$$= 0.362$$

when $i = 4$

$$u_{4,3} = 0.6088 + 0.5(1 - 2(0.6088) + 0.3152)$$
$$= 0.6576$$

at $j = 3$

$$u_{i,4} = u_{i,3} + 0.5(u_{i+1,3} - 2u_{i,3} + u_{i-1,3})$$

when $i = 1$

$$= 0.0576 + 0.5(0.174 - 2(0.0576) + 0)$$
$$= 0.087$$

when $i = 3$

$$u_{3,4} = 0.362 + 0.5(0.6576 - 2(0.362) + 0.174)$$
$$= 0.4158$$

when $i=2$

$$U_{2,4} = U_{2,3} + r(U_{3,3} - 2U_{2,3} + U_{1,3})$$
$$= 0.174 + 0.5(0.362 - 2(0.174) + 0.0576)$$
$$= 0.2098$$

when $i=4$

$$U_{4,4} = U_{4,3} + r(U_{5,3} - 2U_{4,3} + U_{3,3})$$
$$= 0.6576 + 0.5(1 - 2(0.6576) + 0.362)$$

$$U_{4,4} = 0.681$$

at $j=4; i=1$

$$U_{1,5} = 0.087 + 0.5(0.2098 - 2(0.087) + 0)$$
$$= 0.1049$$

$$U_{2,5} = 0.2514$$

when $i=3$

$$U_{3,5} = 0.4158 + 0.5(0.681 - 2(0.4158) + 0.2098)$$
$$= 0.4454$$

when $i=4$

$$U_{4,5} = 0.681 + 0.5(1 - 2(0.681) + 0.4158)$$
$$= 0.7079$$

TABLE

$\frac{r}{k}$	0	0.2	0.4	0.6	0.8	1
0	0	0.0016	0.0256	0.1296	0.4096	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.06	0	0.0576	0.194	0.362	0.6176	1
0.08	0	0.087	0.2098	0.4158	0.681	1
1	0	0.1049	0.2314	0.4454	0.7079	1

	x^i	0	0.2	0.4	0.6	0.8	1	Δt	Δx	r
		i								
t	j	0	1	2	3	4	5			
0	0	0	0.0016	0.0256	0.1296	0.4096	1	0.02	0.2	0.5
0.02	1	0	0.0128	0.0856	0.2176	0.5648	1	0.02	0.2	0.5
0.04	2	0	0.0328	0.1152	0.3152	0.6088	1	0.02	0.2	0.5
0.06	3	0	0.0576	0.174	0.362	0.6576	1	0.02	0.2	0.5
0.08	4	0	0.087	0.2088	0.4158	0.681	1	0.02	0.2	0.5
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1	0.02	0.2	0.5

GRAPH OF TEMPERATURE DISTRIBUTION

