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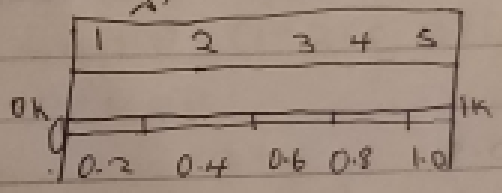
Using Dupuit forward difference to solve the model:

Initial condition $U(x,0) = x^4 K = F(x)$; for $0 \leq x \leq 1m$
for initial condition: $U(t,0) = 0K$ and $U(1,t) = 1K$
Taking assumptions $i = i=0, j=0, K=0, \therefore t=0$
Graphically, we have



Assumptions
The function used is "x", since its a boundary condition
$U(,0) = x^4 K$
- When $x = 0.1m$; $U(1,0) = 0.2^4 = 0.0016$
- When $x = 0.4m$; $U(2,0) = 0.4^4 = 0.0256$
- When $x = 0.6m$; $U(3,0) = 0.6^4 = 0.1296$
- When $x = 0.8m$; $U(4,0) = 0.8^4 = 0.4096$
- When $x = 1m$; $U(5,0) = 1^4 = 1$

Using Fukers method in order to obtain the temp within the Condition gradient:

Recall;
$U_{i,j+1} = r U_{i-1,j} + (1-2r) U_{i,j} + r U_{i+1,j}$ — (*)

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = c\left[\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}\right] \quad \text{---(**)}$$

$$u_{i,j+1} - u_{i,j} = \frac{c\Delta t}{\Delta x^2}\left[u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\right]$$

But $\frac{c\Delta t}{\Delta x^2} = r$, $r = 1 \times 0.02/(0.2)^2 = 0.5$; $r = 0.5$

Evaluating u at $t = 0.02$, $j = 0$, $i = 1, 2, 3, 4$

$u(0,1) = 0$

At $i = 1$

From eqn (*) and $r = 0.5$

$u_{1,0+1} = 0.5u_{1-0,0} + (1 - 2(0.5)) \times 0.0016 + 0.5(0.0256)$

$u_1 = 0.5u(0,0) + 0 + 0.0128 = 0 + 0 + 0.0128$

$u_{1,1} = 0.0128$

At $i = 2$

$u_{2,0+1} = 0.5(u_{1,0}) + (1 - 2(0.5)) \times u_{2,0} + ru_{3,0}$

$= 0.5(0.0016) + (1-1) \times (0.0256) + 0.5(0.1296) = 0.0008 + 0 + 0.0648$

$u_{2,1} = 0.0656$

At $i = 3$

$u_{3,0+1} = ru_{2,0} + (1 - 2r)u_{3,0} + ru_{4,0} = 0.5(0.0256 +$

$(1 - 2(0.5))0.1296 + 0.5(0.4096) = 0.128 + 0 + 0.2048$

$u_{3,1} = 0.2176$

At $i = 4$

$u_{4,1} = r u_{3,0} + (1-2r) u_{4,0} + r u_{5,0}$

$= 0.5(0.1296) + (1 - 2(0.5))(0.4096) + 0.5(1) = 0.0648 + 0 + 0.5$

$u_{4,1} = 0.5648$

Evaluating u at $t = 0.04, j = 1$

At $i = 1$

$u_{i,j+1} = r u_{0,j} + (1-r) u_{i,j} + r u_{2,j}$

$u_{1,1+1} = 0.5(0) + (1 - 2(0.5))(0.0656) + 0.5(0.0656)$

$= 0 + 0 + 0.035$

$u_{1,2} = 0.0328$ //

At $i = 2$

$u_{2,1+j} = r u_{1,j} + (1-2r) u_{2,j} + r u_{3,j}$

$= 0.5(0.0128) + (1 - 2(0.5))(0.0656) + 0.5(0.2176)$

$= 0.0064 + 0 + 0 + 0.1088 = 0.1152$ //

At $i = 3$

$u_{3,1+j} = r u_{2,j} + (1-2r) u_{3,j} + r u_{4,j}$

$= 0.5(0.0656) + (1 - 2(0.5))(0.2176) + 0.5(0.5648)$

$= 0.0328 + 0 + 0.2824 = 0.3152$ //

At $i = 4$

$u_{4,1+j} = r u_{3,j} + (1-2r) u_{4,j} + r u_{5,j}$

$u_{4,j} = 0.5(0.2176) + (1 - 2(0.5))(0.5648) + 0.5 \times 1$

$= 0.1088 + 0 + 0.5$

$u_{4,2} = 0.6088$

Evaluating u at $t = 0.06$, $j = 2$

At $i = 1$

$u_{1,3} = r u_{0,2} + (1-2r) u_{1,2} + r u_{2,2} = 0.5(0) + (1-2\times0.5)(0.0328) + 0.5$

$u_{1,3} = 0 + 0 + 0.576 = 0.0576$

At $i = 2$

$u_{2,3} = r u_{1,2} + (1-2r) u_{2,2} + r u_{3,2}$

$= 0.5(0.0328) + (1-2(0.5))(0.1152) + 0.5(0.3152) = 0.01640 + 0.2160$

$u_{2,3} = 0.1749$

At $i = 3$

$u_{3,3} = r u_{2,2} + (1-2r) u_{3,2} + r u_{4,2}$

$u_{3,3} = 0.5(0.1152) + (1-2(0.5))(0.3152) + 0.5(0.6088)$

$u_{3,3} = 0.0576 + 0 + 0.30440 = 0.3620$

At $i = 4$

$u_{4,3} = r u_{3,2} + (1-2r) u_{4,2} + r u_{5,2}$

$= 0.5(0.3152) + (1-2(0.5))(0.6088) + 0.5(0) = 0.5761 + 0 + 0.5$

$= 0.6596$

Evaluating u at $t = 0.08$, $j = 2$

At $i = 1$

$u_{1,4} = r u_{0,3} + (1-2r) u_{1,3} + r u_{2,3} = 0.5(0) + (1-2\times0.5)(0.05$

$+ 0.5 \times 0.11746$

$u_{1,4} = 0 + 0 + 0.087 = 0.0874$

At i = 2

$U_{0,4j} = rU_{1,3}(1-2r)U_{2,3} + rU_{3,3} = 0.5(0.0376) + (1-2(0.5))(0.1740) + 0.5(0)$

$U_{2,4} = 0.0288 + 0 + 0.181 = 0.2098$ //

At i = 3

$U_{3,4j} = rU_{2,3}(1-2r)U_{3,3} + rU_{4,3} = 0.5 \times 0.1740 + (1-2\times0.5) \times$

$0.362 + 0.6576 + 0.5(0)$

$U_{3,4} = 0.087 + 0 + 0.3288 = 0.4158$

At i = 4

$U_{4,4j} = rU_{3,3} + (1-2r)U_{4,3} + rU_{5,3} = 0.5 \times 0.362 + (1-2\times0.5)$

$\times 0.6576 + 0.5(0)$

$U_{4,4} = 0.181 + 0 + 0.5 = 0.681$ //

Evaluating U at i = 1, j = 4

At i = 1

$U_{i,1+j} = rU_{0,4} + (1-2r)U_{1,4} + rU_{2,4}$

$= 0.5(0) + (1-2\times0.5)\times0.087 + 0.5 \times 0.2098 = 0 + 0 + 0.1049 = 0.1049$ //

At i = 2

$U_{2,4j} = rU_{1,4} + (1-2r)U_{2,4} + rU_{3,4}$

$= 0.5(0.087) + (1-2(0.5)) + (0.2098) + 0.4158(0.5) = 0.0435 + 0 + 0.2079$

$U_{2,5} = 0.2514$

At i = 3

$U_{3,1+j} = rU_{2,4} + (1-2r)U_{3,4} + rU_{4,4}$

$= 0.5(0.2098) + (1-2(0.5))(0.4158) + 0.5(0.681) = 0.1049 + 0 + 0.34$

$U_{3,5} = 0.4454$

Δx	0.2	0	0.4	0.6	0.8	1
0	0.0016	0	0.1056	0.2976	0.4096	1
0.02	0.0128	0	0.0656	0.2176	0.3456	1
0.04	0.0328	0	0.1152	0.352	0.4896	1
0.06	0.0574	0	0.1740	0.3440	0.4578	1
0.08	0.087	0	0.2048	0.4448	0.693	1
1	0.1049	0	0.2514	0.4454	0.7095	1