

Maths assignment

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17/02/062

Computer programming

$$u_t = c u_{xx} = 0$$
$$\frac{du}{dt} = c \frac{d^2u}{dx^2} = 0$$
$$\frac{du}{dt} = c \frac{d^2u}{dx^2}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = \frac{(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})}{\Delta x^2}$$

$$u_{i,j+1} - u_{i,j} = (\Delta t [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]) \cdot \frac{\Delta t}{\Delta x^2}$$
$$j = 1$$

$$u(x,0) = x^4 k$$

$$\Delta x = 0.2, \Delta t = 0.02 \text{ day}$$

four boundary conditions

$$\text{At } x=0 \rightarrow x^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-9}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$\therefore u_{i,j+1} = u_{i,j} + r [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$$

$$U_{i,t+1} = [U_{i-1,t} + (1-2r)U_{i,t} + rU_{i+1,t}] \quad \text{--- (1)}$$

when $i = 1, j = 0$

$$\begin{aligned} U_{1,1} &= 0.5U_{0,0} + 0.5U_{2,0} \\ &= 0.5(0) + 0.5(0.0256) \\ U_{1,1} &= 0.0128 \end{aligned}$$

when $i = 2, j = 0$

$$\begin{aligned} U_{2,1} &= 0.5(U_{1,0}) + 0.5(U_{3,0}) \\ &= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296) \end{aligned}$$

$$U_{2,1} = 0.0656$$

when $i = 3, j = 0 = U_{3,1} = 0.5(U_{2,0}) + 0.5(U_{4,0})$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$U_{3,1} = 0.2176$$

when $i = 4, j = 0 = U_{4,1} = 0.5(U_{3,0}) + 0.5(U_{5,0})$

$$= 0.5(0.256) + 0.5(0.4096)$$

$$U_{4,1} = 0.5648$$

for $j = 1$

when $i = 1 \therefore U_{1,2} = 0.5(U_{2,1})$

$$= 0.5U_{2,1} + 0.5U_{0,1} = 0 + 0.5(0.0656)$$

$$U_{1,2} = 0.0328$$

$$\begin{aligned}\text{When } i=2 : U_{2,2} &= 0.5(U_{1,1}) + 0.5(U_{3,1}) \\ &= 0.5(0.0128) + 0.5(0.2176) \\ U_{2,2} &= 0.1152\end{aligned}$$

$$\begin{aligned}\text{When } i=3 : U_{3,2} &= 0.5(U_{2,1}) + 0.5(U_{4,1}) \\ &= 0.5(0.0656) + 0.5(0.5648) \\ U_{3,2} &= 0.3152\end{aligned}$$

$$\begin{aligned}\text{When } i=4 : U_{4,2} &= 0.5(U_{3,1}) + 0.5(U_{5,1}) \\ &= 0.5(0.2176) + 0.5(1) \\ U_{4,2} &= 0.6088\end{aligned}$$

For $T=2$

$$\begin{aligned}\text{When } i=1 : U_{1,3} &= 0.5(U_{0,1}) + 0.5(U_{2,2}) \\ &= 0 + 0.5(0.1152) = 0.0576\end{aligned}$$

$$\begin{aligned}\text{When } i=2 : U_{2,3} &= 0.5(U_{1,2}) + 0.5(U_{3,2}) \\ &= 0.5(0.0328) + 0.5(0.3152) \\ &= 0.174\end{aligned}$$

$$\begin{aligned}\text{When } i=3 & \therefore U_{3,3} = 0.5(U_{2,2}) + 0.5(U_{4,2}) \\ & = 0.5(0.1152) + 0.5(0.608) \\ & = 0.362\end{aligned}$$

$$\begin{aligned}\text{When } i=4 & U_{4,3} = 0.5(U_{3,2}) + 0.5(U_{2,1}) \\ & = 0.5(0.3152) + 0.5(1) \\ U_{4,3} & = 0.6576\end{aligned}$$

for $j=3$

$$\begin{aligned}\text{When } r=1 & U_{1,4} = 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ & = 0.5(0) + 0.5(0.174) \\ & = 0.087\end{aligned}$$

$$\begin{aligned}\text{When } i=2 & U_{2,4} = 0.5(U_{1,3}) + 0.5(U_{3,3}) \\ & = 0.5(0.0576) + 0.5(0.362) \\ & = 0.2098\end{aligned}$$

$$\begin{aligned}\text{When } i=3 & U_{3,4} = 0.5(U_{2,3}) + 0.5(U_{4,3}) \\ & = 0.5(0.174) + 0.5(0.6576) \\ & = 0.4158\end{aligned}$$

$$\begin{aligned} \text{When } i = 4 \quad U_{4,4} &= 0.5(U_{3,3}) + 0.5(U_{4,0}) \\ &= 0.5(0.362) + 0.5(1) \\ &= 0.681 \end{aligned}$$

For $j = 4$

$$\begin{aligned} \text{When } i = 1 \quad U_{1,5} &= 0.5(U_{0,4}) + 0.5(U_{1,4}) \\ &= 0.5(0.2098) = 0.1049 \end{aligned}$$

$$\begin{aligned} \text{When } i = 2 \quad U_{2,5} &= 0.5(U_{1,4}) + 0.5(U_{3,4}) \\ &= 0.5(0.087) + 0.5(0.468) \\ &= 0.2514 \end{aligned}$$

$$\begin{aligned} \text{When } i = 3 \quad U_{3,5} &= 0.5(U_{2,4}) + 0.5(U_{4,4}) \\ &= 0.5(0.2098) + 0.5(0.681) \\ &= 0.4454 \end{aligned}$$

$$\begin{aligned} \text{When } i = 4 \quad U_{4,5} &= 0.5(U_{3,4}) + 0.5(U_{5,4}) \\ &= 0.5(0.4158) + 0.5(1) \\ &= 0.7079 \end{aligned}$$

Δx	$\frac{1}{h} \text{ km}(k)$							
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1	1
0.08	4	0	0.087	0.2098	0.4158	0.681	1	1
0.06	3	0	0.0676	0.174	0.312	0.468	1	1
0.04	2	0	0.0328	0.1152	0.3152	0.608	1	1
0.02	1	0	0.0128	0.0686	0.2126	0.5645	1	1
0	0	0	0.0016	0.028	0.1296	0.4076	1	1
Δx		0	0.2	0.4	0.6	0.8	1	1
	i	0	1	2	3	4	5	5



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0.1	0	0.1049	0.2514	0.4454	0.7079	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0.0016	0.0256	0.1296	0.4096	1
	0	0.2	0.4	0.6	0.8	1

0.5

