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17 Eng 02 052

Computer Engineering

ENG 582

Assignment 5

$$U_t = (U_{xx} = 0$$

$$\frac{dx}{dt} = \frac{d^2 U}{dx^2} = 0$$

$$\frac{dx}{dt} = \frac{d^2 U}{dx^2}$$

$$U_{i,j+1} - U_{i,j} = \frac{(U_{i+1,j} - 2U_{i,j} + U_{i-1,j})}{\Delta x^2}$$

$$U_{i,j+1} - U_{i,j} = \left[\frac{\Delta t}{\Delta x^2} (U_{i+1,j} - 2U_{i,j} + U_{i-1,j}) \right] \cdot \tau = \frac{\Delta t}{\Delta x^2} \cdot L \cdot \tau$$

$$U[x, 0] = x^2$$

$$\Delta x = 0.2m \quad \Delta t = 0.001s$$

for initial condition.

$$\Delta t \ x = 0 \rightarrow x^2 = 0$$

$$\Delta t \ x = 0.2 \rightarrow 0.2^2 = 0.04$$

$$\Delta t \ x = 0.4 \rightarrow 0.4^2 = 0.16$$

$$\Delta t \ x = 0.6 \rightarrow 0.6^2 = 0.36$$

$$\Delta t \ x = 0.8 \rightarrow 0.8^2 = 0.64$$

$$\Delta t \ x = 1 \rightarrow 1^2 = 1$$

$$\therefore U_{i,j+1} = U_{i,j} + \left[\frac{\Delta t}{\Delta x^2} (U_{i+1,j} - 2U_{i,j} + U_{i-1,j}) \right]$$

$$U_{i,j+1} = \tau (U_{i-1,j}) + (1-2\tau) U_{i,j} + \tau (U_{i+1,j}) - x$$

when $i = 1, j = 0$

$$U_{1,1} = 0.5U_{0,0} + 0.5U_{2,0}$$

$$= 0.5(0) + 0.5(0.16)$$

$$U_{1,1} = 0.08$$

when $i=2, j=0$

$$U_{2,1} = 0.5(U_{2,0}) + 0.5(U_{2,2}) \\ = 0.5(14.0) + 0.5(0.1296) \\ U_{2,1} = 0.0656$$

when $i=3, j=0$

$$U_{3,1} = 0.5(U_{3,0}) + 0.5(U_{3,2}) \\ = 0.5(0.0384) + 0.5(0.4076) \\ U_{3,1} = 0.2176$$

$$\text{when } i=4, j=0 \quad U_{4,1} = 0.5(U_{4,0}) + 0.5(U_{4,2}) \\ = 0.5(0.0384) + 0.5(0.4076) \\ U_{4,1} = 0.2176$$

For $j=1$

$$\text{when } i=3 \quad U_{3,2} = 0.5(U_{3,1}) + 0.5(U_{3,3}) \\ = 0.5(U_{3,1}) + 0.5(U_{3,1}) + 0 + 0.5(0.0664) \\ U_{3,2} = 0.0628$$

$$\text{when } i=2 \quad U_{2,2} = 0.5(U_{2,1}) + 0.5(U_{2,3}) \\ = 0.5(0.0128) + 0.5(0.2176) \\ U_{2,2} = 0.1152$$

$$\text{when } i=5 \quad U_{5,2} = 0.5(U_{5,1}) + 0.5(U_{5,3}) \\ = 0.5(0.0156) + 0.5(0.5648) \\ U_{5,2} = 0.3452$$

$$\text{when } i=4 \quad U_{4,2} = 0.5(U_{4,1}) + 0.5(U_{4,3}) \\ = 0.5(0.2176) + 0.5(1) \\ U_{4,2} = 0.6088$$

For $j=2$

$$\text{when } i=1 \quad U_{1,2} = 0.5(U_{1,1}) + 0.5(U_{1,3}) \\ = 0 + 0.5(0.1152) = 0.0576$$

$$\text{when } i=2 \quad U_{2,3} = 0.5(U_{2,2}) + 0.5(U_{2,4}) \\ = 0.5(0.0528) + 0.5(0.3152) = 0.194$$

$$\text{when } i=5 \quad U_{5,3} = 0.5(U_{5,2}) + 0.5(U_{5,4}) \\ = 0.5(0.052) + 0.5(0.6088) = 0.362$$

0.8375
= 1.125
= 0.7375

U_{4,5}
0.5(0.8375) + 0.5(1) = 0.91875
→ 0.91875

when $i = 4$ $U_{4,5} = 0.5(U_{4,4}) + 0.5(U_{4,6})$
 $0.5(0.9152) + 0.5(1)$
 $U_{4,5} = 0.9576$

for $i = 3$

when $i = 1$ $U_{1,4} = 0.5(U_{1,3}) + 0.5(U_{1,5})$
 $0.5(0) + 0.5(0.174) = 0.087$

when $i = 2$ $U_{2,4} = 0.5(U_{2,3}) + 0.5(U_{2,5})$
 $0.5(0.087) + 0.5(0.362) = 0.2098$

when $i = 3$ $U_{3,4} = 0.5(U_{3,3}) + 0.5(U_{3,5})$
 $0.5(0.174) + 0.5(0.6576) = 0.4158$

when $i = 4$ $U_{4,4} = 0.5(U_{4,3}) + 0.5(U_{4,5})$
 $0.5(0.362) + 0.5(1)$
 $= 0.681$

for $i = 4$

when $i = 1$ $U_{1,5} = 0.5(U_{1,4}) + 0.5(U_{1,6})$
 $= 0.5(0.2098) = 0.1049$

when $i = 2$ $U_{2,5} = 0.5(U_{2,4}) + 0.5(U_{2,6})$
 $0.5(0.087) + 0.5(0.4158) = 0.2514$

when $i = 3$ $U_{3,5} = 0.5(U_{3,4}) + 0.5(U_{3,6})$
 $0.5(0.2098) + 0.5(0.681)$
 $= 0.4454$

when $i = 4$ $U_{4,5} = 0.5(U_{4,4}) + 0.5(U_{4,6})$
 $0.5(0.4158) + 0.5(1)$
 $= 0.7077$

Δt	j		i				
	Temp (°C)	0	1	2	3	4	5
0.1	5	0	0.1049	0.2514	0.4454	0.7077	1
0.08	4	0	0.087	0.2098	0.4158	0.681	1
0.06	3	0	0.2576	0.174	0.362	0.6576	1
0.04	2	0	0.0328	0.1152	0.3152	0.6008	1
0.02	1	0	0.0128	0.0556	0.2176	0.5648	1
0	0	0	0.0016	0.028	0.1296	0.4096	1
Δx		0	0.2	0.4	0.8	1	
		0	1	2	3	4	5

0.1	0	0.1049	0.2514	0.4454	0.7079	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0.0016	0.0256	0.1296	0.4096	1
	0	0.2	0.4	0.6	0.8	1

0.5

