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17/ENG01/030

Chemical Engineering
ENG 382 Assignment

$$u_t - c u_{xx} = 0 \quad \text{for } 0 \leq x \leq 1$$

$$\frac{du}{dt} - \frac{cd^2u}{dx^2} = 0$$

$$du/dt = cd^2u/dx^2$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = \frac{c \cdot u_0 + c u_j - 2u_0 u_j + u_{0-1} j}{\Delta x^2}$$

$$u_{i,j+1} - u_{i,j} = \frac{c \Delta t}{\Delta x^2} [u_0 + u_j - 2u_0 u_j + u_{0-1} j]$$

$$u(x, 0) = x^4$$

$$\Delta x = 0.2 \text{ m}, \quad \Delta t = 0.02 \text{ s}$$

for initial conditions:

$$\text{At } x = 0 \rightarrow x^4 = 0$$

$$\text{At } x = 0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x = 0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x = 0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x = 0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x = 1 \rightarrow 1^4 = 1$$

$$\therefore u_{i,j+1} = u_{i,j} + r [u_0 + u_j - 2u_0 u_j + u_{0-1} j]$$

$$u_{i,j+1} = r u_{0-1} j + (1 + 2r) u_{i,j} + r u_0 + u_j \quad \text{--- (1)}$$

For $i=1$ when $j=1$ $j=0$

$$u_{1,1} = 0.5 u_{0,0} + 0.5 u_{2,0}$$

$$= 0.5(0) + 0.5(0.0256)$$

$$u_{1,1} = 0.0128$$

when $i = 2, j = 0$

$$u_{211} = 0.5(u_{110}) + 0.5(u_{310})$$
$$= 0.5(1.6 \times 10^{-2}) + 0.5(0.1296)$$

$$u_{211} = 0.0656$$

when $i = 3, j = 0$

$$u_{311} = 0.5(u_{210}) + 0.5(u_{410})$$
$$= 0.5(0.0256) + 0.5(0.4096)$$

$$u_{311} = 0.2176$$

when $i = 4, j = 0$

$$u_{411} = 0.5(u_{310}) + 0.5(u_{510})$$
$$= 0.5(0.0256) + 0.5(0.4096)$$

$$u_{411} = 0.5648$$

For $j = 2$

when $j = 1, i = 1$

$$u_{112} = 0.5(u_{011}) + 0.5(u_{211})$$
$$= 0.5 u_{011} + 0.5 u_{211} = 0 + 0.5(0.656)$$

$$u_{112} = 0.0328$$

when $j = 1, i = 2$

$$u_{212} = 0.5(u_{111}) + 0.5(u_{311})$$
$$= 0.5(0.0128) + 0.5(0.2176)$$

$$u_{212} = 0.1152$$

when $j = 1, i = 3$

$$u_{312} = 0.5(u_{211}) + 0.5(u_{411})$$
$$= 0.5(0.0656) + 0.5(0.5648)$$

$$u_{312} = 0.3152$$

when $j = 1, i = 4$

$$u_{412} = 0.5(u_{311}) + 0.5(u_{511})$$
$$= 0.5(0.2176) + 0.5(1)$$

$$u_{412} = 0.6088$$

for $j=2$

$$\begin{aligned}\text{when } i=1 \rightarrow U_{112} &= 0.5(U_{011}) + 0.5(U_{212}) \\ &= 0 + 0.5(0.1152) \\ &= 0.0576\end{aligned}$$

$$\begin{aligned}\text{when } i=2 \rightarrow U_{212} &= 0.5(U_{112}) + 0.5(U_{312}) \\ &= 0.5(0.0576) + 0.5(0.3152) \\ &= 0.174\end{aligned}$$

$$\begin{aligned}\text{when } i=3 \rightarrow U_{312} &= 0.5(U_{112}) + 0.5(U_{412}) \\ &= 0.5(0.1152) + 0.5(0.6088) \\ &= 0.362\end{aligned}$$

$$\begin{aligned}\text{when } i=4 \rightarrow U_{412} &= 0.5(U_{112}) + 0.5(U_{512}) \\ &= 0.5(0.3152) + 0.5(1) \\ U_{412} &= 0.6576\end{aligned}$$

for $j=3$

$$\begin{aligned}\text{when } i=1 \rightarrow U_{113} &= 0.5(U_{013}) + 0.5(U_{213}) \\ &= 0.5(0) + 0.5(0.174) \\ &= 0.087\end{aligned}$$

$$\begin{aligned}\text{when } i=2 \rightarrow U_{213} &= 0.5(U_{113}) + 0.5(U_{313}) \\ &= 0.5(0.087) + 0.5(0.362) \\ &= 0.2245\end{aligned}$$

$$\begin{aligned}\text{when } i=3 \rightarrow U_{313} &= 0.5(U_{213}) + 0.5(U_{413}) \\ &= 0.5(0.2245) + 0.5(0.6576) \\ &= 0.44105\end{aligned}$$

$$\begin{aligned}\text{when } i=4 \rightarrow U_{413} &= 0.5(U_{313}) + 0.5(U_{513}) \\ &= 0.5(0.44105) + 0.5(1) \\ &= 0.720525\end{aligned}$$

for $J = 4$

$$\begin{aligned} \text{when } i=1 \quad u_{115} &= 0.5(u_{014}) + 0.5(u_{214}) \\ &= 0.5(0.2098) + 0 \\ &= 0.1049 \end{aligned}$$

$$\begin{aligned} \text{when } i=2 \quad u_{215} &= 0.5(u_{114}) + 0.5(u_{314}) \\ &= 0.5(0.687) + 0.5(0.7158) \\ &= 0.2514 \end{aligned}$$

$$\begin{aligned} \text{when } i=3 \quad u_{315} &= 0.5(u_{214}) + 0.5(u_{414}) \\ &= 0.5(0.2098) + 0.5(0.687) \\ &= 0.4454 \end{aligned}$$

$$\begin{aligned} \text{when } i=4 \quad u_{415} &= 0.5(u_{314}) + 0.5(u_{514}) \\ &= 0.5(0.4158) + 0.5(1) \\ &= 0.7079 \end{aligned}$$

TABLE

Δt	J	Temp (K)					
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1
0.08	4	0	0.087	0.2098	0.7158	0.681	1
0.06	3	0	0.0576	0.174	0.862	0.5576	1
0.04	2	0	0.0328	0.1152	0.3152	0.6008	1
0.02	1	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0	0.0016	0.028	0.1296	0.4006	1
Δx		0	0.2	0.4	0.6	0.8	1
	i	0	1	2	3	4	5

x/t	0	0.2	0.4	0.6	0.8	1
0	0	0.0016	0.0256	0.1296	0.4096	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.1	0	0.1049	0.2514	0.4454	0.7079	1

e	1				
$r=c*(\Delta t/\Delta x^2)$	0.5				
Δt	0.02				
Δx^2	0.04				

