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17/ENG104/1048 Elect/Elect

ENG 382 ASSIGNMENT 6

$$u_t - C u_{xx} = 0 \quad \text{for } 0 \leq x \leq 1\text{m}, 0 \leq t \leq 0.1 \text{ day}$$
$$\Delta x = 0.2\text{m}$$

$$u_t = C u_{xx}$$

$$u(x,0) = x^4 k$$

$$\text{at } x = 0\text{m and } x = 1\text{m}$$

$$u(0,t) = 0k$$

$$u(1,t) = 1k$$

This is a boundary condition

At $t = 0$, when $x = 0$

$$u_{0,0} = 0^2 k$$

$$= 0$$

$$\text{when } x = 0.2\text{m}$$

substitute

$$u_{1,0} = (0.2)^4 k = 0.0016k$$

$$\text{when } x = 0.4\text{m}$$

$$u_{2,0} = (0.4)^4 k = 0.0256k$$

$$\text{when } x = 0.6\text{m}$$

$$u_{3,0} = (0.6)^4 k = 0.1296k$$

$$\text{when } x = 0.8\text{m}$$

$$u_{4,0} = (0.8)^4 k = 0.4096k$$

$$\text{when } x = 1.0\text{m}$$

$$u_{5,0} = (1)^4 k = 1.0000k$$

Applying Euler method

$$u_{i,j+1} = r u_{i-1,j} + (1-2r) u_{i,j} + r u_{i+1,j}$$

$$\text{Remember } C = 1\text{m}^2/\text{day} \quad \Delta t = 0.02\text{day} \quad \Delta x = 0.2\text{m}$$

where

$$r = \frac{C \Delta t}{\Delta x^2} = \frac{1 \times 0.02}{(0.2)^2} = 0.5$$

Now to get temperature, u

$$u \text{ at } t = 0.02$$

$$j=0 \quad i=1, 2, 3, 4$$

at $i=1$

$$\begin{aligned} u_{1,1} &= r u_{0,0} + (1-2r) u_{1,0} + r u_{2,0} \\ &= 0.5(0) + (1-2(0.5))0.0016 + 0.5(0.0256) \\ &= 0.0128 \end{aligned}$$

at $i=2$

$$\begin{aligned} u_{2,1} &= r u_{1,0} + (1-2r) u_{2,0} + r u_{3,0} \\ &= 0.5(0.0016) + (1-2(0.5))0.0256 + 0.5(0.1296) \\ &= 0.0656 \end{aligned}$$

at $i=3$

$$\begin{aligned} u_{3,1} &= r u_{2,0} + (1-2r) u_{3,0} + r u_{4,0} \\ &= 0.5(0.0256) + (1-2(0.5))0.1296 + 0.5(0.4096) \\ &= 0.2176 \end{aligned}$$

at $i=4$

$$\begin{aligned} u_{4,1} &= r u_{3,0} + (1-2r) u_{4,0} + r u_{5,0} \\ &= 0.5(0.1296) + (1-2(0.5))0.4096 + 0.5(1) \\ &= 0.5648 \end{aligned}$$

u at $t = 0.04, j=1, i=1, 2, 3, 4$

at $i=1$

$$\begin{aligned} u_{1,2} &= r u_{0,1} + (1-2r) u_{1,1} + r u_{2,1} \\ &= 0.5(0) + (1-2(0.5))0.0128 + 0.5(0.0656) \\ &= 0.0328 \end{aligned}$$

at $i=2$

$$\begin{aligned} u_{2,2} &= r u_{1,1} + (1-2r) u_{2,1} + r u_{3,1} \\ &= 0.5(0) + (1-2(0.5))0.0656 + 0.5(0.2176) \\ &= 0.1152 \end{aligned}$$

at $i=3$

$$\begin{aligned}U_{3,2} &= rU_{2,1} + (1-2r)U_{3,1} + rU_{4,1} \\ &= 0.5(0.0652) + (1-2(0.5))(0.2176) + 0.5(0.56) \\ &= 0.3152\end{aligned}$$

at $i=4$

$$\begin{aligned}U_{4,2} &= rU_{3,1} + (1-2r)U_{4,1} + rU_{5,1} \\ &= 0.5(0.2176) + (1-2(0.5)) \cdot 0.5648 + 0.5(1) \\ &= 0.6088\end{aligned}$$

U at $t=0.06$

$j=2 \quad i=1,2,3,4$

at $i=1$

$$\begin{aligned}U_{1,3} &= rU_{0,2} + (1-2r)U_{1,2} + rU_{2,2} \\ &= 0.5(0) + (1-2(0.5))(0.0328) + 0.5(0.1152) \\ &= 0.0576\end{aligned}$$

at $i=2$

$$\begin{aligned}U_{2,3} &= rU_{1,2} + (1-2r)U_{2,2} + rU_{3,2} \\ &= 0.5(0.0328) + (1-2(0.5))(0.1152) + 0.5(0.3152) \\ &= 0.1740\end{aligned}$$

at $i=3$

$$\begin{aligned}U_{3,3} &= rU_{2,2} + (1-2r)U_{3,2} + rU_{4,2} \\ &= 0.5(0.1152) + (1-2(0.5))(0.3152) + 0.5(0.60) \\ &= 0.3620\end{aligned}$$

at $i=4$

$$\begin{aligned}U_{4,3} &= rU_{3,2} + (1-2r)U_{4,2} + rU_{5,2} \\ &= 0.5(0.3152) + (1-2(0.5))(0.6088) + 0.5(1) \\ &= 0.6576\end{aligned}$$

U at $t = 0.08$ $j=3$ $i=1,2,3,4$

at $i=1$

$$\begin{aligned}U_{1,t} &= rU_{0,3} + (1-2r)U_{1,3} + rU_{2,3} \\ &= 0.5(0) + (1-2(0.5))(0.0576) + 0.5(0.1740) \\ &= 0.087\end{aligned}$$

at $i=2$

$$\begin{aligned}U_{2,t} &= rU_{1,3} + (1-2r)U_{2,3} + rU_{3,3} \\ &= 0.5(0.0576) + (1-2(0.5))(0.1740) + 0.5(0.3620) \\ &= 0.2098\end{aligned}$$

at $i=3$

$$\begin{aligned}U_{3,t} &= rU_{2,3} + (1-2r)U_{3,3} + rU_{4,3} \\ &= 0.5(0.1740) + (1-2(0.5))(0.3620) + 0.5(0.6576) \\ &= 0.4158\end{aligned}$$

at $i=4$

$$\begin{aligned}U_{4,t} &= rU_{3,3} + (1-2r)U_{4,3} + rU_{5,3} \\ &= 0.5(0.3620) + (1-2(0.5))(0.6576) + 0.5(1) \\ &= 0.6810\end{aligned}$$

U at $t = 0.1$, $j=4$ and $i=1,2,3,4$

at $i=1$

$$\begin{aligned}U_{1,t} &= rU_{0,4} + (1-2r)U_{1,t} + rU_{2,t} \\ &= 0.5(0) + (1-2(0.5))(0.087) + 0.5(0.2098) \\ &= 0.1049\end{aligned}$$

at $i=2$

$$\begin{aligned}U_{2,t} &= rU_{1,4} + (1-2r)U_{2,t} + rU_{3,t} \\ &= 0.5(0.087) + (1-2(0.5))(0.2098) + 0.5(0.4158) \\ &= 0.2514\end{aligned}$$

at $i=3$

$$U_{3,5} = rU_{2,4} + (1-2r)U_{3,4} + rU_{4,4}$$
$$= 0.5(0.2098 + (1-2(0.5))0.4158 + 0.5(0.681))$$
$$= 0.4454$$

at $i=4$

$$U_{4,5} = rU_{3,4} + (1-2r)U_{4,4} + rU_{5,4}$$
$$= 0.5(0.4158) + (1-2(0.5))(0.681) + 0.5(1)$$
$$= 0.7079$$

TABULAR FORM

$t \backslash x$	0	0.2	0.4	0.6	0.8	1
0.00	0	0.016	0.0256	0.1296	0.4096	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.06	0	0.0576	0.1940	0.3620	0.6576	1
0.08	0	0.0870	0.2098	0.4158	0.6810	1
0.10	0	0.1049	0.2514	0.4454	0.7079	1