Dexter Henry Chikomiso MAT 104.

16/ENG02/038 (Carry over).

Computer Engineering

Differentiate the following.

1.) $y = [(x+1)^2 (x-2)^{1/2}] / [(2x-1)(x+3)^{3/2}]$

2.) $y = [3e^x \sin 2x] / [x^{5/2}]$

Integrate the following with respect to the variable.

1.) $\int \sec^2 (3m+1)$

2.) $2t(3t^2-1)^{1/2}$

3.) $2x/(4x^2-1)^{1/2}$

Solution.

2.) $y = \frac{3e^x \sin 2x}{x^{5/2}}$

$\ln y = \ln 3e^x + \ln \sin 2x - \ln x^{5/2}$

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(3e^x) + \frac{d}{dx}(\ln \sin 2x) - \frac{d}{dx}(\ln x^{5/2})$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{3e^x}(3e^x) + \frac{1}{\sin 2x}(\cos 2x) - \frac{1}{x^{5/2}}(5/2\, x^{3/2})$

$\frac{1}{y}\frac{dy}{dx} = \frac{3e^x}{3e^x} + \frac{\cos 2x}{\sin 2x} - \frac{5/2\, x^{3/2}}{x^{5/2}}$

Multiply both sides by y.

$\frac{1}{y}\frac{dy}{dx} x y = y\left(1 + \frac{\cos 2x}{\sin 2x} - \frac{5/2\, x^{3/2}}{x^{5/2}}\right)$

$\therefore \frac{dy}{dx} = \frac{3e^x \sin 2x}{x^{5/2}}\left(1 + \cot 2x - \frac{5/2\, x^{3/2}}{x^{5/2}}\right)$

Integration

1.) $\int \sec^2 (3m+1)\, dm$

$u = 3m+1$

$du = 3dm$

$dm = \frac{du}{3}$

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$\int 4\sec^2 u \frac{du}{3}$

$\frac{4}{3}\int \sec^2 u\,du$

$\frac{4}{3}\tan u + C$

$\frac{4}{3}\tan(3m+1) + C$

2) $\int 2t\,(3t^2-1)^{1/2}$

$u = 3t^2 - 1$

$\frac{du}{dt} = 6t$

$dt = \frac{du}{6t}$

$\int (u)^{1/2} \frac{du}{3}$

$\int u^{1/2} du$

$\frac{1}{3} \times \frac{u^{1/2+1}}{1/2+1} + C$

$= \frac{1}{3} \times \frac{u^{1/2+1}}{1/2+1} + C$

$= \frac{1}{3} \times \frac{2}{3} u^{3/2} + C$

$= \frac{2}{9} u^{3/2} + C$

$= \frac{2}{9}(3t^2-1)^{3/2} + C$

3) $\int \frac{2x}{(4x^2-1)^{1/2}}$

$u = 4x^2 - 1$

$du = 8x\,dx$

$dx = \frac{du}{8x}$

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$$\int 2x(u)^{-1/2}\frac{du}{8x}$$

$$=\frac{1}{4}\int u^{-1/2}\,du$$

$$=\frac{1}{4}\times\frac{u^{-1/2+1}}{-1/2+1}$$

$$=\frac{1}{4\times}\;\frac{u^{1/2}}{1/2}$$

$$=\frac{1}{4}\times 2u^{1/2}$$

$$=\frac{1}{2}u^{1/2}$$

$$=\frac{1}{2}(4x^2-1)^{1/2}$$