

James Evidence

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Mechanical Engineering

Maths Assignment

$$U_t - C U_{xx} = 0$$

$$\frac{dU}{dt} - C \frac{d^2 U}{dx^2} = 0$$

$$\frac{dU}{dt} = C \frac{d^2 U}{dx^2}$$

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta t} = \frac{C \cdot U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{\Delta x^2}$$

$$U_{i,j+1} - U_{i,j} = \frac{C \Delta t}{\Delta x^2} [U_{i,j+1} - 2U_{i,j} + U_{i,j-1}]$$

$$U[x, 0] = x^4 k$$

$$\Delta x = 0.2 \text{ m}, \Delta t = 0.02 \text{ day}$$

for initial conditions

$$\text{At } x=0 \rightarrow x^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$\therefore U_{i,j+1} = U_{i,j} + r [U_{i,j+1} - 2U_{i,j} + U_{i,j-1}]$$

$$U_{i,j+1} = r U_{i,j+1} + (1-2r) U_{i,j} + r U_{i,j-1}$$

When $i=1, j=0$

$$U_{1,1} = 0.5 U_{0,0} + 0.5 U_{2,0}$$

$$= 0.5(0) + 0.5(0.0256)$$

$$U_{1,1} = 0.0128$$

When $i=2, j=0$

$$U_{2,1} = 0.5(U_{1,0}) + 0.5(U_{3,0})$$

$$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$U_{2,1} = 0.0656$$

$$\text{When } i=3, j=0 \Rightarrow U_{3,1} = 0.5(U_{2,0}) + 0.5(U_{4,0}) \\ = 0.5(0.0256) + 0.5(0.4096) \\ U_{3,1} = 0.2176$$

$$\text{When } i=4, j=0 \Rightarrow U_{4,1} = 0.5(U_{3,0}) + 0.5(U_{5,0}) \\ = 0.5(0.0256) + 0.5(0.4096) \\ = 0.5648$$

for $j=1$

$$\text{When } i=1 \Rightarrow U_{1,2} = 0.5(U_{0,1}) + 0.5(U_{2,1}) \\ = 0.5(U_{0,1}) + 0.5(U_{2,1}) = 0.5(0.0856) \\ = 0.0328$$

$$\text{When } i=2 \Rightarrow U_{2,2} = 0.5(U_{1,1}) + 0.5(U_{3,1}) \\ = 0.5(0.0128) + 0.5(0.2176) \\ = 0.1152$$

$$\text{When } i=3 \Rightarrow U_{3,2} = 0.5(U_{2,1}) + 0.5(U_{4,1}) \\ = 0.5(0.5648) + 0.5(0.5648)$$

$$\text{When } i=4 \Rightarrow U_{4,2} = 0.5(U_{3,1}) + 0.5(U_{5,1}) \\ = 0.6088$$

for $j=2$

$$\text{When } i=1 \Rightarrow U_{1,3} = 0.5(U_{0,2}) + 0.5(U_{2,2}) \\ = 0 + 0.5(0.1152) = 0.0576$$

$$i=2 \Rightarrow U_{2,3} = 0.5(U_{1,2}) + 0.5(U_{3,2}) \\ = 0.1744$$

$$i=3 \Rightarrow U_{3,3} = 0.5(U_{2,2}) + 0.5(U_{4,2}) \\ = 0.5(0.1152) + 0.5(0.6088) = 0.362$$

$j=3$

$$\text{When } i=1 \Rightarrow U_{1,4} = 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ = 0 + 0.5(0.1744) = 0.0872$$

$$\text{When } i=2 \Rightarrow U_{2,4} = 0.5(U_{1,3}) + 0.5(U_{3,3}) \\ = 0.2098$$

$$\text{When } i=3 \Rightarrow U_{3,4} = 0.5(U_{2,3}) + 0.5(U_{4,3}) \\ = 0.5(0.1744) + 0.5(0.6576) = 0.416$$

$$\text{When } i=4 \Rightarrow U_{4,4} = 0.5(U_{3,3}) + 0.5(U_{5,3}) \\ = 0.5(0.362)$$

for $J=8$

when $i=1$ $u_{1,5} = 0.5(u_{0,4}) + 0.5(u_{1,4})$
 $= 0.5(0.2098) + 0.5(0.6049)$

when $i=2$ $u_{2,5} = 0.5(u_{1,4}) + 0.5(u_{2,4})$
 $= 0.5(0.587) + 0.5(0.4158) = 0.5114$

when $i=3$ $u_{3,5} = 0.5(u_{2,4}) + 0.5(u_{3,4})$
 $= 0.5(0.2098) + 0.5(0.680)$
 $= 0.4454$

when $i=4$ $u_{4,5} = 0.5(u_{3,4}) + 0.5(u_{4,4})$
 $= 0.5(0.4158) + 0.5(0)$
 $= 0.2079$

At	J	k						
0.1	5	0	0.1049	0.2514	0.4454	0.7079		
0.08	4	0	0.587	0.2098	0.4158	0.680		
0.66	3	0	0.576	0.174	0.362	0.571		
0.04	2	0	0.328	0.112	0.3152	0.6008		
0.02	1	0	0.0128	0.065	0.2196	0.5608		
	0	0	0.016	0.028	0.1296	0.6096		
		0	0.2	0.4	0.6	0.8		
		0	1	2	3	4		