

Olavido Arayo Alvar  
 174 En 906 / 084  
 Mechanical Engineering  
 Assignment

$$u_e - C u_{xx} = 0$$

$$\frac{du}{dt} - C \frac{\partial^2 u}{\partial x^2} = 0$$

$$\frac{du}{dt} = C \frac{\partial^2 u}{\partial x^2}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = \frac{C \cdot u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta x^2}$$

$$u_{i,j+1} - u_{i,j} = \frac{C \Delta t}{\Delta x^2} [u_{i,j+1} - 2u_{i,j} + u_{i,j-1}]$$

$$u[x, 0] = x^4 k$$

$$\Delta x = 0.2 \text{ m}, \Delta t = 0.02 \text{ day}$$

for initial conditions

$$\text{At } x=0 \rightarrow x^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$\therefore u_{i,j+1} = u_{i,j} + r [u_{i,j+1} - 2u_{i,j} + u_{i,j-1}]$$

$$u_{i,j+1} + [u_{i,j-1} + (1-2r)u_{i,j} + r u_{i,j+1}]$$

When  $i=1, j=0$

$$u_{1,1} = 0.5 u_{0,0} + 0.5 u_{2,0}$$

$$= 0.5(0) + 0.5(0.0256)$$

$$u_{1,1} = 0.0128$$

When  $i=2, j=0$

$$u_{2,1} = 0.5(u_{1,0}) + 0.5(u_{3,0})$$

$$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$u_{2,1} = 0.0656$$

$$\text{when } i=3, j=0 = U_{3,1} = 0.5(U_{2,0}) + 0.5(U_{4,0}) \\ = 0.5(0.0256) + 0.5(0.4096) \\ U_{3,1} = 0.2176$$

$$\text{when } i=4, j=0 = U_{4,1} = 0.5(U_{3,0}) + 0.5(U_{5,0}) \\ = 0.5(0.0256) + 0.5(0.4096) \\ = 0.2176$$

for  $j=1$

$$\text{when } i=0 = U_{0,2} = 0.5(U_{0,1}) + 0.5(U_{2,1}) \\ = 0.5(U_{0,1}) + 0.5(U_{2,1}) = 0.5(0.0328) \\ = 0.0328$$

$$\text{when } i=2 = U_{2,2} = 0.5(U_{1,1}) + 0.5(U_{3,1}) \\ = 0.5(0.0128) + 0.5(0.2176) \\ = 0.1152$$

$$\text{when } i=3 = U_{3,2} = 0.5(U_{0,1}) + 0.5(U_{4,1}) \\ = 0.5(0.0328) + 0.5(0.2176) \\ = 0.1252$$

$$\text{when } i=4 = U_{4,2} = 0.5(U_{2,1}) + 0.5(U_{5,1}) \\ = 0.2176$$

for  $j=2$

$$\text{when } i=1 = U_{1,3} = 0.5(U_{0,2}) + 0.5(U_{2,2}) \\ = 0 + 0.5(0.1152) = 0.0576$$

$$i=2 = U_{2,3} = 0.5(U_{1,2}) + 0.5(U_{3,2}) \\ = 0.1252$$

$$i=3 = U_{3,3} = 0.5(U_{2,2}) + 0.5(U_{4,2}) \\ = 0.5(0.1252) + 0.5(0.2176) = 0.1714$$

$j=3$

$$\text{when } i=1 = U_{1,4} = 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ = 0 + 0.5(0.0576) = 0.0288$$

$$\text{when } i=2 = U_{2,4} = 0.5(U_{1,3}) + 0.5(U_{3,3}) \\ = 0.0288 + 0.5(0.1714) = 0.0907$$

$$\text{when } i=3 = U_{3,4} = 0.5(U_{2,3}) + 0.5(U_{4,3}) \\ = 0.5(0.0907) + 0.5(0.2176) = 0.1541$$

$$\text{when } i=4 = U_{4,4} = 0.5(U_{3,3}) + 0.5(U_{5,3}) \\ = 0.5(0.1714) + 0.5(0.2176) = 0.1945$$

At  $J=0$

when  $i=1$   $u_{1,5} = 0.5(u_{0,4}) + 0.5(u_{1,4})$   
 $= 0.5(0.2098) + 0.5(0.649)$   
 $= 0.4294$

when  $i=2$   $u_{2,5} = 0.5(u_{1,4}) + 0.5(u_{3,4})$   
 $= 0.5(0.4294) + 0.5(0.8152) = 0.6223$

when  $i=3$   $u_{3,5} = 0.5(u_{2,4}) + 0.5(u_{4,4})$   
 $= 0.5(0.4294) + 0.5(0.681)$   
 $= 0.5547$

when  $i=4$   $u_{4,5} = 0.5(u_{3,4}) + 0.5(u_{5,4})$   
 $= 0.5(0.4294) + 0.5(1)$   
 $= 0.7147$

At	$J_{\text{heap}(k)}$						
0.1	5	0	0.1049	0.2574	0.4454	0.7079	1
0.08	4	0	0.089	0.2098	0.4188	0.681	1
0.66	3	0	0.576	0.194	0.362	0.6576	1
0.04	2	0	0.0328	0.1152	0.5152	0.6058	1
0.02	1	0	0.0128	0.0652	0.2196	0.5816	1
0	0	0	0.016	0.028	0.1296	0.4096	1
0.2		0	0.2	0.4	0.6	0.8	1
		0	1	2	3	4	1