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MATRIC NO.: 19/ENG04/047
COURSE CODE/TITLE: MAT 104 → CALCULUS
20-5-20 SERIAL NO.: 16

1. Evaluate dy/dx at $x = 2.5$, correct to 3 significant figures

given $y = \frac{(2x^2+3)}{\ln 2x}$

Solution.

$$y = \frac{2x^2+3}{\ln 2x}$$

Using quotient rule.

$u = 2x^2 + 3$ $\qquad$ $v = \ln 2x$

$\frac{du}{dx} = 4x$ $\qquad$ $\frac{dv}{dx} = \frac{1}{x}$

$$\frac{dy}{dx} = \frac{v\,du/dx - u\,dv/dx}{v^2}$$

$$= \frac{(\ln 2x)(4x) - (2x^2+3)(\frac{1}{x})}{(\ln 2x)^2}$$

$$= \frac{4x \ln 2x - 2x - 3x^{-1}}{(\ln 2x)^2}$$

$$\frac{dy}{dx} = \frac{4x}{\ln 2x} - \frac{2x}{(\ln 2x)^2} - \frac{3}{x(\ln 2x)^2}$$

At $x = 2.5$,

$$\frac{dy}{dx} = \frac{4(2.5)}{\ln 2(2.5)} - \frac{2(2.5)}{[\ln 2(2.5)]^2} - \frac{3}{2.5[\ln 2(2.5)]^2}$$

$$\frac{dy}{dx} = 6.213349346 - 1.930285505 - 0.463268521$$

$$\frac{dy}{dx} = 3.81979532$$

To 3 s.f.,

$$\frac{dy}{dx} = 3.82.$$

2 Find the gradient of the curve $y = \frac{2x}{x^2-5}$ at the point $(2, -\frac{4}{1})$

Solution.

$y = \frac{2x}{x^2-5}$

using quotient rule,

$u = 2x$ $\quad v = x^2 - 5$

$\frac{du}{dx} = 2$ $\quad \frac{dv}{dx} = 2x$

$\frac{dy}{dx} = \frac{v\, du/dx - u\, dv/dx}{v^2}$

$= \frac{(x^2-5)(2) - (2x)(2x)}{(x^2-5)^2}$

$= \frac{2x^2 - 10 - 4x^2}{(x^2-5)^2}$

$\frac{dy}{dx} = \frac{-2x^2 - 10}{(x^2-5)^2}$ = Gradient

At $x = 2$,

$\frac{dy}{dx} = \frac{-2(2)^2 - 10}{[(2)^2 - 5]^2}$

$= \frac{-18}{1}$

$\frac{dy}{dx} = -18$ = Gradient

3 If $z = 2x^3 \ln y$, find dz/dy.

Solution

$z = 2x^3 \ln y$

$\frac{dz}{dy} = 2x^3 \cdot \frac{d}{dy}(\ln y)$

$\frac{dz}{dy} = 2x^3 \cdot \frac{1}{y} = \frac{2x^3}{y} = 2x^3 y^{-1}$

4 Integrate $x(2x^2+1)^{1/2}$ with respect to x from 0 to 2.

Solution

$$\int_0^2 x(2x^2+1)^{1/2}\,dx$$

Let $v = 2x^2 + 1$

$$\frac{dv}{dx} = 4x$$

$$dv = 4x\,dx$$

$$dx = \frac{dv}{4x}$$

$$\int_0^2 x(2x^2+1)^{1/2}\,dx = \int_0^2 x(v)^{1/2}\cdot\frac{dv}{4x}$$

$$= \int_0^2 \frac{v^{1/2}\,dv}{4}$$

$$= \frac{1}{4}\int_0^2 v^{1/2}\,dv$$

$$= \left[\frac{1}{4}\cdot\frac{v^{1/2+1}}{1/2+1}\right]_0^2$$

$$= \left[\frac{1}{4}\cdot\frac{v^{3/2}}{3/2}\right]_0^2$$

$$= \left[\frac{1}{4}\cdot\frac{2}{3}\cdot v^{3/2}\right]_0^2$$

$$= \left[\frac{1}{6}v^{3/2}\right]_0^2$$

$$= \left[\frac{1}{6}(2x^2+1)^{3/2}\right]_0^2$$

$$= \left[\frac{1}{6}[2(2)^2+1]^{3/2}\right] - \left[\frac{1}{6}[2(0)^2+1]^{3/2}\right]$$

$$= \frac{9}{2} - \frac{1}{6}$$

$$\int_0^2 x(2x^2+1)^{1/2}\,dx = 4.33$$