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Using Explicit forward difference method

$U_{xx} = CU_{xxx}$ for $\therefore 0 \leq x \leq 1m, 0 \leq t \leq 0.1$ day, $\Delta x = 0.2m$

$\therefore U_t = CU_{xxx}$

Initial Condition

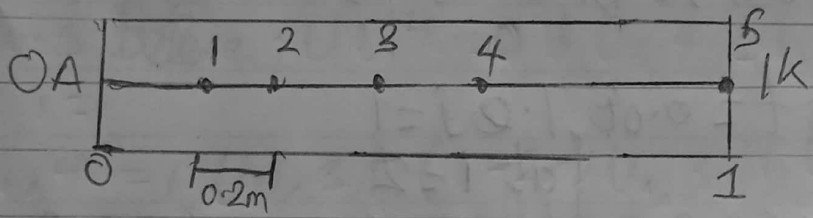
$U(x, 0) = x^4 k = f(x)$

Boundary Condition

$U(0, t) < OR \longrightarrow U(1, t) = 1k$

Graphically

$i = 0, j = 0 \therefore t = 0$



Since it is at Boundary Condition we the function of x at $t = 0$

$U_{a,0} = 0^\circ_A$

When $x = 0.2m$
$U_{1,0} = (0.2)^4$
$= 0.0016$

When $x = 0.4m$
$U_{2,0} = (0.4)^4$
$= 0.0256$

When $x = 0.6m$,
$U_{3,0} = (0.6)^4$
$= 0.1296$

When $x = 0.8$
$U_{4,0} = (0.8)^4$
$= 0.4016$

When $x = 1$
$U_{5,0} = (1)^4$
$= 1$

To find the temperature within the condition gradient, we use the discritized foward Eular method

$U_{i,j+1} = rU_{i-1,j} + (1-2s)U_{i,j} + rU_{i+1,j}$

Where

$r = \frac{\Delta t}{\Delta x^2} = 1 \times \frac{0.02}{(0.2)^2} = 0.5$

Evaluating U at $t = 0.02$ i.e $j = 0$, $i = 1, 2, 3, 4$

$U_{0,1} = 0$ (BC) $\quad U_{5,1} = 1$ (BC)

at $i = 1$

$U_{1,1} = rU_{2,0} + (1-2r)U_{0,0} + rU_{0,0}$

$= 0.5(0) + (1-2(0.5))0.0016 + 0.5(0.0330)$

$= 0 + 0 + 0.128$

$= 0.0128$

at $i = 2$

$U_{2,1} = rU_{1,0} + (1-2r)U_{2,0} + rU_{3,0}$

$= 0.5(0.0016) + (0)(0.0256) + 0.5(0.06) = 0.0656$

at $i = 3$

$U_{3,1} = rU_{4,0} + (1-2r)U_{3,0} + rU_{2,0}$

$= 0.5(0.0256) + (1-2(0.5))(0.1296) + 0.5(0.4096)$

$= 0.2176$

at $i = n$

$U_{n,1} = rU_{3,0} + (1-2r)U_{n,0} + rU_{5,0}$

$= 0.5(0.1296) + (1-2(0.5))(0.4096) + 0.5(1) = 0.5648$

$\therefore U_{1,1}, U_{2,1}, U_{3,1}; U_{4,1} = 0.0128, 0.0656, 0.2176, 0.5648$

Evaluating U at $t = 0.04$, i.e $j = 1$

at $i = 1$

$U_{1,2} = rU_{0,1} + (1-2r)U_{1,1} + rU_{2,1}$

$= 0.5(0) + (0)(0.0128) + 0.5(0.0656) = 0.0328$

at $i = 2$

$U_{2,2} = rU_{1,1} + (1-2r)U_{2,1} + rU_{3,1}$

$= 0.5(0.0127) + (0)(0.0656) + 0.5(0.2196)$

$= 0.1152$

at $i = 3$

$U_{3,2} = rU_{2,1} + (1-2r)U_{3,1} + rU_{4,1}$

$= 0.5(0.0656) + 0(0.2176) + 0.5(0.5648)$

$= 0.3152$

at $i = 4$

$U_{n,2} = rU_{3,1} + (1-2r)U_{n,1} + rU_{5,1}$

$= 0.5(0.2176) + 0(0.5648) + 0.5(1) = 0.6088$

N/B:- $U_{0,2} = 0$ & $U_{5,2} = 1$ (due to Boundary condition)

Evaluating U at $t = 0.06$ i.e $j = 2$

at i=1

$U_{1,3} = rU_{0,2} + (1-2r)U_{1,2} + rU_{2,2}$

$= 0.5(0) + 0(0.0328) + 0.5$

$= (0.1152)$

$= 0.0576$

at i=2

$U_{2,3} = rU_{1,2} + (1-2r)U_{2,2} + rU_{3,2}$

$= 0.5(0.0328) + (0)(0.1152) + 0.5(0.3152)$

$= 0.1740$

at i=3

$U_{3,3} = rU_{2,2} + (1-2r)U_{3,2} + rU_{4,2}$

$= 0.5(0.1152) + (0)(0.3152) + 0.5(0.6088)$

$= 0.3620$

at i=4

$U_{4,3} = rU_{3,2} + (1-2r)U_{4,2} + rU_{3,2}$

$= 0.5(0.3152) + 0(0.6088) + 0.5(1)$

$= 0.6596$

N/B = $U_{0,3} = 0$ & $U_{3,2} = 1$

Evaluating U at t = 0.08 i & j = 3

Notes before: $U_{0,4} = 0$ & $U_{3,4} = 1$

at i=1

$U_{1,4} = rU_{0,3} + (1-2r)U_{1,3} + rU_{2,3}$

$= 0.5(0) + 0(0.0576) + 0.5(0.1740) = 0.087$

at i=2

$U_{2,1} = rU_{1,3} + (1-2r)U_{2,3} + rU_{3,3}$

$= 0.5(0.0576) + 0(0.1740) + 0.5(0.362)$

$= 0.2098$

at i=3

$U_{3,n} = rU_{2,3} + (1-2r)U_{3,5}$

$+ 0.5(0.174) + 0(0.362)$

$+ 0.5(0.6596)$

$= 0.4158$

at i=4

$U_{3,5} = rU_{2,4} + (1-2r)U_{3,4} + rU_{4,4}$

$= 0.5(0.2098) + 0(0.4158) + 0.5(0.681) = 0.4454$

at i=2

$U_{2,5} = rU_{1,4} + (1-2r)U_{2,4} + rU_{3,4}$

$= 0.5(0.087) + 0(0.2097) + 0.5(0.4158) = 0.2514$

at i=4

$U_{n,5} = rU_{3,4} + (1-2r)U_{n,4} + rU_{4,4}$

$= 0.5(0.2098) + 0(0.4158) + 0.5(0.681) + 0.5(1) = 0.7079$

NIB $U_{0,2}=0$ $U_{5,5}=1$

t \ x	0	0.2	0.4	0.6	0.8	2
0	0	0.0016	0.0256	0.1296	0.4096	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0328	0.1152	0.3132	0.6088	1
0.06	0	0.0576	0.194	0.362	0.6376	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.1	0	0.1049	0.2514	0.4459	0.7079	1