Using the Following :

sin(x)sin(y)=1/2(cos(y−x)−cos(y+x)), sin\*\*2(x)=1/2(1−cos(2x)),  
cos(x)cos(y)=1/2(cos(y+x)+cos(y−x)), cos\*\*2(x)=1/2(cos(2x)+1),  
sin(x)cos(y)=1/2(sin(y+x)−sin(y−x)), cos(x)sin(x)=1/2sin(2x)

1. ∫cos(2x)sin(7x)dx

=∫(sin(9x)+sin(5x))/2dx

=1/2∫sin(9x)dx+1/2∫sin(5x)dx

∫sin(9x)dx

Substitute u=9x

=1/9∫sin(u)du

∫sin(u)du

=−cos(u)

1/9∫sin(u)du

=−cos(u)/9

=−cos(9x)/9

∫sin(5x)dx

Sub u=5x

=1/5∫sin(u)du

∫sin(u)du

=−cos(u)

1/5∫sin(u)du

=−cos(u)5

subs u=5x:

=−cos(5x)/5

1/2∫sin(9x)dx+1/2∫sin(5x)dx

=−cos(9x)/18−cos(5x)/10

= −(cos(9x)/18)−(cos(5x)/10)+C

1. ∫Cos3xCosxdx

=∫sin((9x)+sin(5x))/2dx

=1/2∫sin(9x)dx+1/2∫sin(5x)dx

∫sin(9x)dx

Substitute u=9x

=1/9∫sin(u)du

∫sin(u)du

=−cos(u)

1/9∫sin(u)du

=−cos(u)/9

=−cos(9x)/9

∫sin(5x)dx

Substitute u=5x

=1/5∫sin(u)du

∫sin(u)du

=−cos(u)

1/5∫sin(u)du

=−cos(u)/5

=−cos(5x)/5

1/2∫sin(9x)dx+1/2∫sin(5x)dx

=−(cos(9x)/18)−(cos(5x)/10)

=−(cos(9x)/18)−(cos(5x)/10)+C

1. ∫cos(x)/sin(2x)dx

−ln(|csc(x)+cot(x)|)/2+C

∫cos(x)/sin(2x)dx

=∫csc(x)/2dx

=1/2∫csc(x)dx

∫csc(x)dx

Expand fraction by csc(x)+cot(x)

=∫csc(x)(csc(x)+cot(x))/csc(x)+cot(x)dx

=∫csc^2(x)+cot(x)csc(x)/csc(x)+cot(x)dx

Sub u=csc(x)+cot(x)

=−∫1/udu

∫1/udu

=ln(u)

−∫1/udu

=−ln(u)

=−ln(csc(x)+cot(x))

12∫csc(x)dx

=−ln(csc(x)+cot(x))/2

=−ln(|csc(x)+cot(x)|)/2+C

1. ∫ 9x^2ydx

[lim 0->3]

=9x^2⋅∫ydy

=y^2/2

9x^2⋅∫ydy

=9x^2y^2/2

=(9x^2y^2/2)+C

Integrating with second limit:

∫ 9x^2y^2/2)+Cdx

[lim 1->2]

=9y^2/2∫x^2dx+c∫1dx

=x^3/3

=x

9y^2/2∫x^2dx+c∫1dx

=3y^2x^3/2+cx

=3y^2x^3/2+cx+C