19/ENG05/048 MAT104 CLASSWORK

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MECHATRONICS ENGINEERING

* Evaluate the following

(4) $\int_1^2 \int_0^3 9x^2y \, dx \, dy$ ~~(2) $\int \cos 5x \cos 6x \, dx$~~ (1) $\int \sin 7x \cos 2x \, dx$

(3) $\int \frac{\cos x}{\sin^2 x} dx$ (2) $\int \cos 3x \cos x \, dx$

Solution:

(4) $\int_1^2 \int_0^3 9x^2y \, dx \, dy$

Considering the inside Integral

$\int_0^3 9x^2 y \, dx$

$= \left[\frac{\cancel{9}^{3} x^3 y}{\cancel{3}}\right]_0^3$

$= \left[3x^3y\right]_0^3$

$= [3(3)^3y] - [0]$

$= [3(27)y]$

$= 81y$

$\therefore \int_1^2 81y \, dy$

$= \left[\frac{81y^2}{2}\right]_1^2$

$= \left[\frac{81(2^2)}{2} - \frac{81(1^2)}{2}\right]$

$= 162 - 40.5$

$= 121.5_u$

$\therefore \int_1^2 \int_0^3 9x^2y \, dx \, dy = 121.5_u$

(1) $\int \cos 5x \cos 6x \, dx$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$A = 5x, \; B = 6x$

$\therefore \cos 5x \cos 6x = \frac{1}{2}[\cos(5x+6x) + \cos(5x-6x)]$

$\cos 5x \cos 6x = \frac{1}{2}[\cos 11x + \cos -x]$

$\therefore \int \cos 5x \cos 6x \, dx = \frac{1}{2}\left[\int \cos 11x + \int \cos -x\right]$

$= \frac{1}{2}\left[\frac{1}{11}\sin 11x + \left(\frac{1}{-1}\sin -x\right)\right]$

$\int \cos 5x \cos 6x \, dx = \frac{1}{22}\sin 11x - \frac{1}{2}\sin -x + C_u$

$\int \cos 5x \cos 6x \, dx = \frac{\sin 11x}{22} - \frac{\sin -x}{2} + C_u$

(1) $\int \sin 7x \cos 2x \, dx$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$A = 7x, \; B = 2x$

$\therefore \sin 7x \cos 2x = \frac{1}{2}[\sin(7x+2x) + \sin(7x-2x)]$

$\sin 7x \cos 2x = \frac{1}{2}[\sin 9x + \sin 5x]$

$\therefore \int \sin 7x \cos 2x \, dx = \frac{1}{2}\left[\int \sin 9x + \int \sin 5x\right]$

$= \frac{1}{2}\left[\frac{-1}{9}\cos 9x - \frac{1}{5}\cos 5x\right] + C$

$\therefore \int \sin 7x \cos 2x \, dx = -\frac{1}{18}\cos 9x - \frac{1}{10}\cos 5x + C$

$\therefore \int \sin 7x \cos 2x \, dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$

(3) $\int \frac{\cos x}{\sin^2 x} dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\therefore \int \frac{\cos x}{\sin^2 x} dx = \int \frac{\cancel{\cos x}}{u^2} \frac{du}{\cancel{\cos x}}$

$= \int \frac{1}{u^2} du$

$= \int u^{-2} du$

$= \left[\frac{u^{-2+1}}{-2+1}\right] + c$

$= \frac{u^{-1}}{-1} + c$

$= -u^{-1} + c$

$= -(\sin x)^{-1} + c$

$\therefore \int \frac{\cos x}{\sin^2 x} dx = -\sin^{-1} x + c$ //

(2) $\int \cos 3x \cos x \, dx$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$A = 3x, \quad B = x$

$\therefore \int \cos 3x \cos x \, dx = \frac{1}{2}[\cos(3x+x) + \cos(3x-x)]$

$\cos 3x \cos x = \frac{1}{2}[\cos 4x + \cos 2x]$

$\therefore \int \cos 3x \cos x \, dx = \frac{1}{2}\left[\int \cos 4x + \int \cos 2x\right]$

$\therefore \int \cos 3x \cos x \, dx = \frac{1}{2}\left[\frac{1}{4}\sin 4x + \frac{1}{2}\sin 2x\right]$

$\therefore \int \cos 3x \cos x \, dx = \frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x + c$ //

$\therefore \int \cos 3x \cos x \, dx = \frac{\sin 4x}{8} + \frac{\sin 2x}{4} + c$ //