At $i=4$

$U_{4,2} = rU_{3,1} + (1-2r)U_{4,1} + rU_{5,1} = 0.5(0.2016) + 0(0.564)$
$+ 0.5(1) = 0.6088$

Evaluating U at $t=0.06$ i.e $j=2$ at i:

at $i=1$

$U_{1,3} = rU_{0,2} + (1-2r)U_{1,2} + rU_{2,2} = 0.5(0) + 0(0.0326) + ($
$= 0.0576$

at $i=2$

$U_{2,3} = rU_{1,2} + (1-2r)U_{2,2} + rU_{3,2} = 0.5(0.0326) + 0$
$(0.1152) + 0.5(0.3152) = 0.1740$

Note $U_{0,3} = 0$ & $U_{5,3} = 1$

at $i=3$

~~Evaluating U at $t=0.08$ i.e $j=3$~~

$U_{3,3} = rU_{2,2} + (1-2r)U_{3,2} + rU_{4,2} = 0.3620$

at $i=4$

$U_{4,3} = rU_{3,2} + (1-2r)U_{4,2} + rU_{5,2} = 0.5(0.3152) +$
$0(0.6088) + 0.5(1) = \cancel{0.66}\ 0.6576$

Evaluating u at $t=0.08$ i.e: $j=3$

Noted before: $U_{0,4} = 0$ & $U_{5,4} = 1$

at $i=1$

$U_{1,4} = rU_{0,3} + (1-2r)U_{1,3} + rU_{2,3} = 0.5(0) + 0(0.0567) +$
$0.5(0.1740) = 0.087$

at $i=2$

$U_{2,4} = rU_{1,3} + (1-2r)U_{2,3} + rU_{3,3} = 0.5(0.0576) +$
$0(0.1740) + (0.5)(0.362) = 0.2098$

at $i=3$

~~at $i=$~~ $U_{3,4} = rU_{2,3} + (1-2r)U_{3,3} + rU_{4,3} = \cancel{0}$
$= 0.5(0.174) + 0(0.362) + 0.5(0.6576) = 0.4158$

at $i=4$

$U_{4,4} = rU_{3,3} + (1-2r)U_{4,3} + rU_{5,3} = 0.5(0.362)$
$+ 0(0.6576) + 0.5(1) = 0.681$

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Evaluating U at t=1, j=4

at i=1

$U_{1,5} = rU_{0,4} + (1-2r)U_{1,4} + rU_{2,4} =$

$0.5(0) + 0(0.087) + 0.5(0.2098) = 0.1049$

at i=2

$U_{2,5} = rU_{1,4} + (1-2r)U_{2,4} + rU_{3,4}$

$= 0.5(0.087) + 0(0.2098) + 0.5(0.4158)$

$= 0.0435 + 0 + 0.2079 = 0.2514$

at i=3

$U_{3,5} = rU_{2,4} + (1-2r)U_{3,4} + rU_{4,4} =$

$0.5(2098) + 0(0.4158) + 0.5(0.681)$

$= 0.4454$

at i=4

$U_{4,5} = rU_{3,4} + (1-2r)U_{4,4} + rU_{5,4} =$

$0.5(0.4158) + 0(0.681) + 0.5(1)$

$= 0.7079$

N/B: $U_{0,5} = 0$ & $U_{5,5} = 1$

t\x	0	0.2	0.4	0.6	0.8	1
0	0	0.0016	0.0256	0.1296	0.4096	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.06	0	0.0576	0.144	0.362	0.6576	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.1	0	0.1049	0.2314	0.4454	0.7079	1

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21/05/2020 ENG 382

AJANI Kehinde Patrick

17/ENG02/005

Petroleum engineering

Q4

Solution

Using Explicit forward difference method

$U_t = U_{xx}$ for $0 \le x \le 1m$, $0 \le t \le 0.1$ day

$\Delta x = 0.2m$

$\therefore U_t = C U_{xx}$

Initial condition

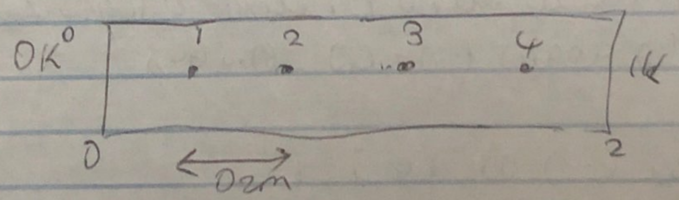
$U(x,0) = x^4 K = f(x)$

Boundary condition

$U(0,t) = 0K \longrightarrow U(1,t) = 1K$

Graphically

$i = 0, j = 0 \therefore t = 0$



Since it is at Boundary Condition, we use the function of x

At $t = 0$

$U_{0,0} = 0°K$

when $x = 0.4m$

$U_{2,0} = (0.4)^4 = 0.0256$

When $x = 0.8$

$U_{4,0} = (0.8)^4 = 0.4096$

when $x = 0.6m$

$U_{3,0} = (0.6)^4 = 0.1296$

When $x = 1$

$U_{5,0} = (1)^4$

$= 1$

$-4 \quad -4x$

The temperature within the condition gradient,
$U_{i,j+1} = r\,U_{i-1,j} + (1-2r)\,U_{i,j} + r\,U_{i+1,j}$
where
$r = \frac{c\Delta t}{\Delta x^2} = \frac{1 \times 0.02}{(0.2)^2} = 0.5$

Evaluating u at $t = 0.02$ i.e $j = 0$, $i = 1,2,3,4$
$U_{0,1} = 0$ (BC)
at $i=1$
$U_{1,1} = rU_{0,0} + (1-2r)U_{0,1} + rU_{2,0} = 0.5(0) + (1-2(0.5))$
$0.0016 + 0.5(0.0230) = 0.0128$

at $i=2$
$U_{2,1} = rU_{1,0} + (1-2r)U_{2,0} + rU_{3,0} = 0.5(0.0016) + 0$
$(0.0256) + 0.5(0.06) = 0.0656$

at $i=3$
$U_{3,1} = rU_{2,0} + (1-2r)U_{3,0} + rU_{4,0} = 0.5(0.0256) +$
$(1-2(0.5))(0.1296) + 0.5(0.4096) = 0.2126$

at $i=4$
$U_{4,1} = rU_{3,0} + (1-2r)U_{4,0} + rU_{5,0} = 0.5(0.1296)$
$+ (1-2(0.5))(0.4096) + 0.5(1) = 0.5648$

Evaluating U at $t = 0.01$ i.e $j = 1$
at $i=1$
$U_{1,2} = rU_{0,1} + (1-2r)U_{1,1} + rU_{2,1} = 0.5(0) + 0 \times (0.0128)$
$+ 0.5(0.0656) = 0.0328$

at $i=2$
$U_{2,2} = rU_{1,1} + (1-2r)U_{2,1} + rU_{3,1} = 0.5(0.0128)$
$+ (0)(0.0656) + 0.5(0.2176) = 0.1152$

at $i=3$
$U_{3,2} = rU_{2,1} + (1-2r)U_{3,1} + rU_{4,1} =$
$0.5(0.0656) + 0(0.2176) + 0.5(0.5648) = 0.3152$