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MAT 104 | Thursday 21st of May 2020
19/ENG06/052

① $\int \sin 7x \cos 2x \, dx =$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\int \sin 7x \cos 2x = \frac{1}{2}[\sin(7x+2x) + \sin(7x-2x)]$

$= \frac{1}{2}[\sin(11x) + \sin(5x)]$

$\int \sin 7x \cos 2x = \frac{1}{2}\left[\int \sin 11x \, dx + \int \sin 5x \, dx\right]$

$= \frac{1}{2}\left[\frac{-1}{11}\cos 11x - \frac{1}{5}\cos 5x\right] + C$

$\frac{-1}{22}\cos 11x - \frac{1}{10}\cos 5x + C$

2 $\cos 3x \cos x \, dx$

Note: $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\int \cos 3x \cos x \, dx = \frac{1}{2}\int [\cos(3x+x) + \cos(3x-x)] dx$

$= \frac{1}{2}\left[\int \cos 4x \, dx + \int \cos 2x \, dx\right]$

$\frac{1}{2}\left[\frac{1}{4}\sin 4x + \frac{1}{2}\cos 2x\right] + C$

$\frac{1}{8}\sin 4x + \frac{1}{4}\cos 2x + C$

(3) $\int \frac{\cos x}{\sin^2 x}\, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\, dx$

$dx = \frac{du}{\cos x}$

$\int \frac{\cos x}{\sin^2 x} \cdot \frac{du}{\cos x}$

$\int \frac{du}{\sin^2 x}$

$\int \frac{du}{u^2}$

$\int u^{-2}\, du$

$\frac{u^{-1}}{-1} + C$

$= -\frac{1}{u} + C$

$= \frac{-1}{\sin x} + C$

(4) $\int_1^2 \int_0^3 9x^2 y\, dx\, dy$

$\int_0^3 9x^2 y\, dx$

$\left[\frac{9x^3 y}{3}\right]_0^3$

$\left[3x^3 y\right]_0^3$

$\int_1^2 81y\, dy$

$\left[\frac{81y^2}{2}\right]_1^2$

$\frac{324}{2} - \frac{81}{2}$

$\frac{243}{2}$