

19/EN905/017

Find the integral of the following

1. $\int \sin 7x \cos 2x \, dx$

Solution

$$\sin A \cos B \, dx = \frac{1}{2} [\sin(A+B) + \sin(A-B)] \, dx \quad A = 7x \quad B = 2x$$

$$\int \sin 7x \cos 2x \, dx = \int \frac{1}{2} [\sin(7x+2x) + \sin(7x-2x)] \, dx$$

$$\int \frac{1}{2} [\sin(9x) + \sin(5x)] \, dx$$

$$\frac{1}{2} \left[-\frac{1}{9} \cos 9x - \frac{1}{5} \cos 5x \right] + C$$

$$\therefore \int \sin 7x \cos 2x \, dx = -\frac{1}{18} \cos 9x - \frac{1}{10} \cos 5x + C$$

2. $\int \cos 3x \cos x \, dx$

Solution

$$\cos A \cos B \, dx = \frac{1}{2} [\cos(A+B) + \cos(A-B)] \, dx \quad A = 3x \quad B = x$$

$$\int \cos 3x \cos x \, dx = \int \frac{1}{2} [\cos(3x+x) + \cos(3x-x)] \, dx$$

$$\int \cos 3x \cos x \, dx = \int \frac{1}{2} [\cos(4x) + \cos(2x)] \, dx$$

$$\frac{1}{2} \left[\frac{1}{4} \sin 4x + \frac{1}{2} \sin 2x \right] + C$$

$$\therefore \int \cos 3x \cos x \, dx = \frac{1}{8} \sin 4x + \frac{1}{4} \sin 2x + C$$

$$3 \int \frac{\cos x}{\sin^2 x} dx$$

solution

$$\int \frac{\cos x}{\sin^2 x} dx$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$
$$dx = \frac{du}{\cos x}$$

$$\int \frac{\cos x}{u^2} \frac{du}{\cos x}$$

$$\int \frac{du}{u^2}$$

$$\frac{u^{-2+1}}{-2+1} + C$$

$$-U^{-1} + C$$

$$\int \frac{\cos x}{\sin^2 x} = -\frac{1}{U} + C$$

$$= -\frac{1}{\sin x} + C$$

$$\int \frac{\cos x}{\sin^2 x} = -\operatorname{cosec} x + C$$

$$4 \int_1^2 \int_0^3 (9x^2y) dx dy$$

$$\left[\frac{9x^3y}{3} \right]_0^3$$

$$\int_1^2 3 \cdot (3)^3 y dy$$

$$\int_1^2 81y dy$$

$$\left[\frac{81y^2}{2} \right]_1^2$$

$$\frac{81(2)^2}{2} - \frac{81(1)^2}{2}$$

$$\frac{81(2)}{1} - \frac{81}{2}$$

$$\frac{324 - 81}{2} = \frac{243}{2} = 121.5$$

$$\therefore \int_1^2 \int_0^3 (9x^2y) dx = 121.5$$