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Mechatronics Engineering
MAT 104
19/ENG05/018

Find the integral of the following

1 $\sin 7x \cos 2x \, dx$

solution

$$\int \frac{\sin 9x + 5x}{2} dx$$

$$\frac{1}{2}\int \sin 9x \, dx + \frac{1}{2}\int \sin 5x \, dx$$

$$\frac{1}{9}\int \sin u \, du = \frac{-\cos u}{9} = \frac{-\cos(9x)}{9}$$

$$\int \sin 5x \, dx = \frac{1}{5}\int \sin(u) du = -\cos(u)$$

$$\frac{1}{5}\int \sin(u) du = \frac{-\cos u}{5} = \frac{\cos 5x}{5}$$

$$\frac{1}{2}\int \sin 9x \, dx + \frac{1}{2}\int \sin 5x \, dx$$

$$= \frac{-\cos 9x}{18} - \frac{\cos(5x)}{10}$$

$$= \int \cos 2x \sin 7x \, dx.$$

$$= \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$$

2 $\cos 3x \cos 5x \, dx$

solution

$$= \int \cos x \cos x \cos(2x) - \sin(x) \sin(2x)$$

$$= \int \cos^2(x) \cos 2x - \cos(x) \sin(x) \sin(2x)$$

$$= \int \cos^2(x) \cos(2x) \, dx - \int \cos(x) \sin x \sin 2x$$

$$= \int \cos^2(x) \cos 2x \, dx$$

$$= \int \left(\frac{\cos 4x + 2\cos 2x}{4} + \frac{1}{4}\right) dx$$

$$= \int \cos 4x \, dx$$

$$= \frac{1}{4}\int \cos u \, du$$

$$= \int \cos u \, du = \sin(u)$$

$\frac{1}{4}\int \cos u \, du$

$= \frac{\sin u}{4}$

$= \frac{\sin 4x}{4}$

$\int \cos 2x \, dx$

$= \frac{1}{2}\int \cos(u) \, du$

$\int \cos(u) \, du$

$= \sin(u)$

$\frac{1}{4}\int \cos(4x) \, dx + \frac{1}{2}\int \cos 2x \, dx + \frac{1}{4}\int 1 \, dx$

$\frac{\sin 4x}{16} + \frac{\sin(2x)}{4} + \frac{x}{4}$

$\int \cos^2(x) \cos 2x \, dx - \int \cos(x) \sin(x) \sin 2x$

$= \frac{\sin 4x}{8} + \frac{\sin 2x}{4}$

$\int \cos x \cos 3x \, dx$

$\frac{\sin 4x}{8} + \frac{\sin 2x}{4} + C$

3 $\cos x / \sin^2 x \, dx$

solution

$\frac{-x}{\sin(x)} - \int -\frac{1}{\sin(x)} \, dx$

$= \int -\csc(x) \, dx$

$= \ln(\csc(x) + \cot(x))$

$= -\int \csc(x) \, dx + \cot(x))$

$-\int \csc(x) \, dx$

$\ln \csc(x) + \cot(x)$

$= -\frac{x}{\sin(x)} - \int -\frac{1}{\sin(x)} \, dx$

$= \ln(\csc(x) + \cot(x)) - \frac{x}{\sin x}$.