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MATHS 104.

Find the integral of the following:

1. $\int \sin 7x \cos 2x \, dx$

$A = 7x \quad B = 2x$

$\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$

$= \frac{1}{2}\left[\sin(7x+2x) + \sin(7x-2x)\right]$

$= \frac{1}{2}\left[\sin 9x + \sin 5x\right]$

$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\int(\sin 9x + \sin 5x)$

$= \frac{1}{2}\left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right]$

$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$

$\therefore \int \sin 7x \cos 2x \, dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$

2. $\int \cos 3x \cos x \, dx$

$A = 3x \quad B = 1x$

$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$= \frac{1}{2}\left(\cos(3x+1x) + \cos(3x-1x)\right)$

$= \frac{1}{2}\left[\cos 4x + \cos 2x\right]$

$\int \cos 3x \cos x \, dx = \frac{1}{2}\int(\cos 4x + \cos 2x)$

$= \frac{1}{2}\left[\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right]$

$= \frac{1}{2}\left[\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right]$

$= \frac{\sin 4x}{8} + \frac{\sin 2x}{4} + C$

$\therefore \int \cos 3x \cos x \, dx = \frac{\sin 4x}{8} + \frac{\sin 2x}{4} + C$

3. $\int \frac{\cos x}{\sin^2 x} \, dx$

3. $\int \frac{\cos x}{\sin^2 x} dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos$

$dx = \frac{du}{\cos x}$

$\therefore \cos^2 x = 1 - \sin^2 x$

$\int \frac{\cos x \cdot \cos^2 x}{u^2} \cdot \frac{du}{\cos x}$

$\int \frac{1 - \sin^2 x}{u^2} = \int \frac{1 - u^2}{u^2} du$

$\int \left(\frac{1}{u^2} - \frac{u^2}{u^2} \right) du$

$\int (u^{-2}) du$

$= \frac{u^{-1}}{-1} + C \Rightarrow \frac{u^{-1}}{-1}$

$= \frac{(\sin x)^{-1}}{-1}$

$= -\sin x^{-1}$

$\therefore \int \frac{\cos x}{\sin^2 x} dx = -\sin x^{-1}$ or $\frac{-1}{\sin x}$

4.

$$\int_1^2 \left[\int_0^3 9x^2 y \, dx \right] dy$$

$$\int_0^3 9x^2 y \, dx$$

$$\left[\frac{9x^3 y}{3} \right]_0^3 \Rightarrow 81y$$

$$\int_1^2 81y \, dy$$

$$= \left[\frac{81y^2}{2} \right]_1^2$$

$$= \frac{81(2)^2}{2} - \frac{81(1)^2}{2}$$

$$= \frac{324}{2} - \frac{81}{2} = \frac{243}{2} = 121.5$$