NWOKORHE PASCAL
19/ENG05/043
MECHATRONICS ENGINEERING.
MATHS 104.

Find the integral of the following:

1. $\int \sin 7x \cos 2x \, dx$

$A = 7x \quad B = 2x$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$= \frac{1}{2}[\sin(7x+2x) + \sin(7x-2x)]$

$= \frac{1}{2}[\sin 9x + \sin 5x]$

$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right]$

$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$

$\therefore \int \sin 7x \cos 2x \, dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$

2. $\int \cos 3x \cos x \, dx$

$A = 3x \quad B = 1x$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$= \frac{1}{2}[\cos(3x+1x) + \cos(3x-1x)]$

$= \frac{1}{2}[\cos 4x + \cos 2x]$

$\int \cos 3x \cos x \, dx = \frac{1}{2}[(\cos 4x + \cos 2x)$

$= \frac{1}{2}\left[\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right]$

$= \frac{1}{2}\left[\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right]$

$= \frac{\sin 4x}{8} + \frac{\sin 2x}{4} + C$

$\therefore \int \cos 3x \cos x \, dx = \frac{\sin 4x}{8} + \frac{\sin 2x}{4} + C$

3. $\int \frac{\cos x}{\sin^2 x} \, dx$

Scanned with CamScanner

3. $\int \frac{\cos x \, dx}{\sin^2 x}$

Let $u = \sin x$

$\frac{du}{dx} = \cos$

$dx = \frac{du}{\cos x}$

$\therefore \cos^2 x = 1 - \sin^2 x$

$\int \frac{\cos x \cdot \cos^2 x}{u^2} \cdot \frac{du}{\cos x}$

$\int \frac{1 - \sin^2 x}{u^2} = \int \frac{1 - u^2}{u^2} du$

$\int \left(\frac{1}{u^2} - \frac{u^2}{u^2} \right) du$

$\int (u^{-2}) \, du$

$= \frac{u^{-1}}{-1} + C \Rightarrow \frac{u^{-1}}{-1}$

$= \frac{(\sin x)^{-1}}{-1}$

$= -\sin x^{-1}$

$\therefore \int \frac{\cos x \, dx}{\sin^2 x} = -\sin x^{-1}$ or $\frac{-1}{\sin x}$

Scanned with CamScanner

(4.) $\int_1^2 \left[\int_0^3 9x^2y \, dx\right] dy$

$\int_0^3 9x^2y \, dx$

$\left[\frac{9x^3y}{3}\right]_0^3 \Rightarrow 81y$

$\int_1^2 81y \, dy$

$= \left[\frac{81y^2}{2}\right]_1^2$

$= \frac{81(2)^2}{2} - \frac{81(1)^2}{2}$

$= \frac{324}{2} - \frac{81}{2} = \frac{243}{2} = 121.5$

Scanned with CamScanner