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Questions

Find the Integral of the following (i) $\sin 7x \cos 2x\,dx$ (ii) $\cos 3x \cos x\,dx$ (iii) $\cos x / \sin^2 x\,dx$ (iv) double Integral with limits from 1 to 2, from 0 to 3 $(9x^2y)\,dx\,dy$.

Answer

(1) $\int \sin 7x \cos 2x\,dx$ $A = 7x$ $B = 2x$

$\sin A \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$

$= \frac{1}{2}[\sin(7x+2x) + \sin(7x-2x)]$

$= \frac{1}{2}[\sin 9x + \sin 5x]$

$\int \sin 7x \cos 2x\,dx = \frac{1}{2}\int(\sin 7x + \sin 5x)$

$= \frac{1}{2}\left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5}\right]$

$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$

$\int \sin 7x \cos 2x\,dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$ //

(2) $\int \cos 3x \cos x\,dx$ $A = 3x$ $B = 1x$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$= \frac{1}{2}[\cos(3x+1x) + \cos(3x-1x)]$

$= \frac{1}{2}[\cos 4x + \cos 2x]$

$\int \cos 3x \cos x\,dx = \frac{1}{2}\int(\cos 4x + \cos 2x)$

$= \frac{1}{2}\left[\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right]$

$= \frac{1}{2}\left[\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right]$

$= \frac{\sin 4x}{8} + \frac{\sin 2x}{4} + C$

$\int \cos 5x \cos x\,dx = \frac{\sin 4x}{8} + \frac{\sin 2x}{4} + C$

3) $\cos x / \sin^2 x \, dx$

Solution

$$\int \frac{\cos x}{\sin^2 x} dx \qquad \text{let } U = \sin x \quad \frac{dU}{dx} = \cos$$

$$dx = \frac{dU}{\cos x} \qquad \cos^2 x = 1 - \sin^2 x$$

$$\int \frac{\cos^2 x \cancel{\cos x}}{U^2} \frac{dU}{\cancel{\cos x}}$$

$$\int \frac{1 - \sin^2 x}{U^2} = \int \frac{1 - U^2}{U^2} dU$$

$$\int \left(\frac{1}{U^2} - \frac{U^2}{U^2}\right) dU$$

$$\int (U^{-2}) dU = \frac{U^{-1}}{-1} + C \Rightarrow \frac{U^{-1}}{-1} = \frac{(\sin x)^{-1}}{-1}$$

$$\int \frac{\cos x}{\sin^2 x} dx = -\sin x^{-1} \text{ or } \frac{-1}{\sin x} \qquad = -\sin x^{-1}$$

4) double integral with limits from 1 to 2, from 0 to 3 $(9x^2y) dx dy$

Solution

$$\int_1^2 \left[\int_0^3 9x^2 y \, dx\right] dy$$

$$\int_0^3 9x^2 y \, dx$$

$$\left[\frac{9x^3 y}{3}\right]_0^3 \Rightarrow 81y$$

$$\int_1^2 81y \, dy$$

$$= \left[\frac{81y^2}{2}\right]_1^2$$

$$= \frac{81(2)^2}{2} - \frac{81(1)^2}{2} = \frac{324}{2} - \frac{81}{2} = \frac{243}{2} = 121.5 //$$