$\int \sin 7x \cos 2x \, dx$

Solution

Recall $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$A = 7x$ and $B = 2x$

$\int \sin 7x \cos 2x \, dx = \int \frac{1}{2}[\sin(7x+2x) + \sin(7x-2x)]$

$= \int \frac{1}{2}[\sin 9x + \sin 5x]$

$= \frac{1}{2}[-\frac{1}{9}\cos 9x + \frac{1}{5}\cos 5x]$

$= -\frac{1}{18}\cos 9x + \frac{1}{10}\cos 5x$

$\int \cos 3x \cos x \, dx$

Recall $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$A = 3x$ and $B = x$

$\int \cos 3x \cos x \, dx = \int \frac{1}{2}[\cos(3x+x) + \cos(3x-x)]$

$= \int \frac{1}{2}[\cos 4x + \cos 2x]$

$= \frac{1}{2}[\frac{1}{4}\sin 4x + \frac{1}{2}\sin 2x]$

$= \frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x$

$\int \frac{\cos x}{\sin^2 x} \, dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x \quad dx = \frac{du}{\cos x}$

$\therefore \int \frac{\cos x}{\sin^2 x} \, dx = \int \frac{\cos x}{u^2} \cdot \frac{du}{\cos x} = \int \frac{1}{u^2} \, du$

$= \int u^{-2} \, du$

$= \frac{u^{-2+1}}{-2+1} + C = \frac{u^{-1}}{-1} + C = -\frac{1}{u} + C$

$= -\frac{1}{\sin x} + C$

$\int_1^2 \int_0^3 9x^2 y \, dx \, dy$

Solution

$\int_0^3 9x^2 y \, dx$

$= \left[\frac{9x^3 y}{3}\right]_0^3 dx$

$= [3x^3 y]_0^3$

$= [3(3)^3 y] - [3(0)^3 y]$

$= 81y$

$\therefore \int_1^2 81y \, dy$

$\left[\frac{81y^2}{2}\right]_1^2$

$= \left[\frac{81(2)^2}{2}\right] - \left[\frac{81(1)^2}{2}\right]$

$= 162 - 40.5$

$= 121.5$