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19/MHS01/393
Medicine and surgery
MAT 104

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Assignment

(1) Integrate the following functions

(1) $2x^2 \ln x$

Solution → $\int 2x^2 \ln x \, dx$

$u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$ $\therefore du = \frac{1}{x} dx$

$dv = 2x^2 dx$ $\therefore$ $v = \frac{2x^3}{3}$

$\int u\,dv = uv - \int v\,du$

$= \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{1}{x} dx$

$= \frac{2x^3}{3} \ln x - \frac{1}{3} \int 2x^2 dx$

$= \frac{2x^3}{3} \ln x - \frac{1}{3} \cdot \frac{2x^3}{3} + c$

$= \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + c$

$\cancel{\therefore \int 2x^2 \ln x \, dx = \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + c}$

$\cancel{\therefore \int 2x^2 \ln x \, dx = 2\left[\frac{x^3}{3} \ln x - \frac{x^3}{9}\right] + c}$

$= \frac{2x^3}{3}\left(\ln x - \frac{1}{3}\right) + c$

$\therefore \int 2x^2 \ln x \, dx = \left[\frac{2x^3}{3}\left(\ln x - \frac{1}{3}\right) + c\right]$

(2) $\int 3te^{2t}\,dt$

$= u = 3t \quad , \frac{du}{dt} = 3 \quad \therefore du = 3dt$

$= dv = e^{2t} \quad , v = \int e^{2t}dt = \frac{1}{2}e^{2t}$

$= \int u\,dv = uv - \int v\,du$

$= 3t \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \cdot 3dt$

$= \frac{3te^{2t}}{2} - \frac{3}{2}\int e^{2t}dt$

$= \frac{3}{2}te^{2t} - \frac{3}{2} \cdot \frac{e^{2t}}{2} + C$

$= \frac{3}{2}te^{2t} - \frac{3}{4}e^{2t} + C$

$\therefore \int 3te^{2t}dt = \left[\frac{3}{2}te^{2t} - \frac{3e^{2t}}{4}\right] + C$

(3) $\int x^2 \sin x\,dx$

$u = x^2 \quad \therefore \frac{du}{dx} = 2x \quad , du = 2x\,dx$

$dv = \sin x\,dx \quad , v = -\cos x$

$\therefore \int u\,dv = uv - \int v\,du$

$= x^2 \cdot (-\cos x) - \int (-\cos x) \cdot 2x\,dx$

$= -x^2\cos x - \int -2x\cos x\,dx$

$= -x^2\cos x + 2\left[\int x\cos x\,dx\right]$

$= -x^2\cos x + 2\left[x\sin x + \cos x\right] + C$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

$\therefore \int x^2 \sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + C$

(4) $\int \cos 5x \cos 6x \, dx$

$A = 5x \quad , \; B = 6x$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right] \cancel{dx}$$

$$= \frac{1}{2}\left[\cos 11x + \cos(-x)\right] \cancel{dx}$$

$$= \frac{1}{2}\left[\cos 11x - \cos x\right] \cancel{dx}$$

$$\int \cos 5x \cos 6x \, dx = \frac{1}{2}\int (\cos 11x - \cos x)\, dx$$

$$= \frac{1}{2}\left[\frac{\sin 11x}{11} - \frac{\sin x}{1}\right]$$

$$= \frac{\sin 11x}{22} - \frac{\sin x}{2} + C$$

$$\therefore \int \cos 5x \cos 6x \, dx = \frac{\sin 11x}{22} - \frac{\sin x}{2} + C$$

(5) $\int \sin 7x \cos 2x$

$A = 7x \quad , \; B = 2x$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$= \frac{1}{2}\left[\sin(9x) + \sin(5x)\right]$$

$$\int \sin 7x \cos 2x = \frac{1}{2}\int (\sin 9x + \sin 5x)\, dx$$

$$= \frac{1}{2}\left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5}\right]$$

$$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$

$$\therefore \int \sin 7x \cos 2x = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$