1. $\int \sin(7x)\cos(3x)\,dx$

$\sin$

$\sin A \cdot \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$

$A = 7x$

$B = 2x$

$\sin(7x)\cos(2x) = \frac{1}{2}\int \sin(9x)\,dx + \frac{1}{2}\int \sin(4x)\,dx$

$= \frac{1}{2}\left[-\cos\left(\frac{10x}{10}\right)\right] + \frac{1}{2}\left[\left(\frac{-\cos(4x)}{4}\right)\right] + C$

$= -\frac{1}{20}\cos(10x) - \frac{1}{8}\cos(4x) + C$

2. $\int \cos(5x)\cos(6x)$

$\cos A \cdot \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\cos 5x \cos 6x = \frac{1}{2}[\cos(11x) + \cos(-x)]$

$a = 5x$

$b = 6x$

$\int \cos 5x \cos 6x\,dx = \int \frac{1}{2}\cos 11x + \cos(-x)\,dx$

$\text{recall} = \int \cos a \cdot x\,dx = \frac{\sin ax}{a}$

$= \frac{1}{2}\left(\frac{\sin 11x}{11} + \frac{\sin(-x)}{-1}\right.$

$= \frac{\sin(11x)}{121} + \frac{\sin(-x)}{1} + C$ //

3. $\int x^2 \sin(x)\,dx$

$\int u\,dv = uv - \int v\,du$

let $u = x^2 = du = 2x\,dx$

$dv = \sin(x)\,dx = V = -\cos(x)$

then

$I = -x^2\cos x + \int \cos(x)\,2x\,dx$

$= -x^2\cos x + 2\int x\cos(x)\,dx$

$u = x,\ dv = \cos(x)\,dx$

$du = dx$

$V = \sin(x)$

$= -x^2\cos x + 2(x\sin(x)) - \int \sin x\,dx$

$= -x^2\cos x + 2x\sin(x) + 2\cos(x) + C_{11}$

$= -x^2\cos x + 2x\sin(x)$

4. $\int 2x^2 \ln x\,dx$

$= \int u\,dv = uv - \int v\,du$

$u = \ln(x) \rightarrow du = 1/x\,dx$

$dv = 2x^2 \rightarrow V = \frac{2x^3}{3}$

$= \ln(x)\cdot\frac{2x^3}{3} - \int \frac{2x^3}{3}\cdot 1/x\,dx = \frac{2x^3}{3}\ln(x) - 1/3(2x^2\,dx)$

$= \frac{2x^3}{3}\ln(x) - 1/3\cdot\frac{2x^3}{3} = \frac{2x^3}{3}\cdot\ln(x) - \frac{2x^3}{9} + C$

Ans $= \frac{2x^3}{3}\cdot\ln(x) - \frac{2x^3}{9} + C_4$

5. $3te^{2t}$

$$\int 3te^{2t}\,dx = 3t\frac{e^{2t}}{2} - \int 3\cdot\frac{e^{2t}}{2}\,dt$$

$$= \tfrac{1}{2}\,3te^{2t} - \tfrac{3}{2}\int e^{2t}\,dt$$

$$\int 3te^{2t}\,dt = \tfrac{1}{2}\,3te^{2t} - \tfrac{3}{2}\left(e^{2t}/2\right) + \int 2t\,\frac{e^{2t}}{2}\,dt\Big]$$

$$= \tfrac{1}{2}\,3te^{2t} + \tfrac{3}{4}e^{2t} - \tfrac{3}{2}\int e^{2t}\,dt$$

$$\int 3te^{2t}\,dt = \tfrac{1}{2}\,3te^{2t} + \tfrac{3}{4}e^{2t} + \tfrac{3}{2}\left[e^{2t}/2\right] - \int 1\,\frac{e^{2t}}{2}\,dt$$

$$= \tfrac{1}{2}\,3te^{2t} + \tfrac{3}{4}e^{2t} + \tfrac{3}{4}e^{2t} - \tfrac{3}{4}\left(\frac{e^{2t}}{2}\right) + c$$

$$= \tfrac{1}{2}\,3te^{2t} + \tfrac{3}{4}e^{2t} + \tfrac{3}{4}e^{2t} + \tfrac{3}{8}e^{2t} + c$$