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(1) $\int \sin 7x \cos 2x \, dx$

Solution

$\int \sin 7x \cos 2x \, dx$

Recall

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$A = 7x \quad B = 2x$

$\sin A \cos B = \frac{1}{2}[\sin(7x+2x) + \sin(7x-2x)$

$= \frac{1}{2}[\sin 9x + \sin 5x]$

$\int \sin 7x \cos 2x \, dx = \int \frac{1}{2}[\sin 9x + \sin 5x]$

$= \frac{1}{2}[\sin 9x + \sin 5x]$

$\int \sin 7x \cos 2x \, dx = \int \frac{1}{2}[\sin 9x + \sin 5x]$

$= \frac{1}{2}\int [\sin 9x + \sin 5x$

$= \frac{1}{2}\left[\int \sin 9x + \int \sin 5x\right.$

$= \frac{1}{2}\left[\frac{-\cos 9x}{9} + \left(\frac{-\cos 5x}{5}\right)\right]$

$= \frac{1}{2}\left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right] + C$

$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$

2. $\cos 3x \cos x \, dx$

Solution

$\int \cos 3x \cos x \, dx$

Recall

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$A = 3x$ and $B = x$

$\cos A \cos B = \frac{1}{2}[\cos(3x+x) + \cos(3x-x)]$

$= \frac{1}{2}[\cos 4x + \cos 2x]$

$\int \cos 3x \cos x \, dx = \int \frac{1}{2}[\cos 4x + \cos 2x]$

$= \frac{1}{2}(\cos 4x + \cos 2x$

$= \frac{1}{2}[\int \cos 4x + \int \cos 2x]$

$= \frac{1}{2}\left[\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right] + C$

$= \frac{\sin 4x}{8} + \frac{\sin 2x}{4} + C$

(3) $\frac{\cos x}{\sin^2 x} \, dx$

$\int \frac{\cos x}{\sin^2 x} \, dx$

Let $v = \sin x$

$\frac{dv}{dx} = \cos x$

$dv = dx \cos x$

$dx = \frac{1}{\cos x} \, dv$

We have

$\int \frac{\cos x}{\sin^2 x} \, dx = \int \frac{\cos x}{\sin^2 x} \times \frac{1}{\cos x} \, dv = \int \frac{1}{\sin^2 x} \, dv$

recall, $v = \sin x$

$= \int \frac{1}{v^2} \, dv = \int v^{-2} \, dv$

$= \left[\frac{v^{-2+1}}{-2+1}\right] = \frac{v^{-1}}{-1} = -v^{-1}$

$= \frac{-1}{v} = -1/\sin x$

(4) $\int_1^2$

④ $\int_1^2 \left(\int_0^3 9x^2 y \, dx \right) dy$

Solution

$\int_0^3 9x^2 y \, dx$

$= \left[3x^3 y \right]_0^3 = 3(3)^3 y - 3(0)^3 y$

$= 81y$

$= \int_1^2 81y \, dy$

$= \left[\frac{81y^2}{2} \right]_1^2 = \frac{81(2)^2}{2} - \frac{81(1)^2}{2}$

$= \frac{324}{2} - \frac{81}{2} = \frac{324 - 81}{2} = \frac{243}{2}$

$= 121.5$