

① Find the integral for the following

a) $\int \sin 7x \cos 2x \, dx$

b) $\int \cos 3x \cos x \, dx$

c) $\int \cos x / \sin^2 x \, dx$

solution

a) $\int \sin 7x \cos 2x \, dx = \int \frac{1}{2} [\sin(A+B) + \sin(A-B)]$

$$= \int \frac{1}{2} [\sin(7x+2x) + \sin(7x-2x)] = \frac{1}{2} \int [\sin(7x+2x) + \sin(7x-2x)]$$

$$= \frac{1}{2} \left[\frac{\sin 9x}{9} + \frac{\sin 5x}{5} \right] = \frac{1}{2} \left[\frac{\sin 9x}{9} + \frac{\sin 5x}{5} \right] + C$$

$$= \frac{1}{2} \left[-\frac{\cos 9x}{9} + \frac{\cos 5x}{5} \right] + C$$

② $\int \cos 3x \cos x \, dx = \int \frac{1}{2} [\cos(A+B) + \cos(A-B)]$

$$= \frac{1}{2} \int [\cos(3x+x) + \cos(3x-x)] = \frac{1}{2} \int [\cos 4x + \cos 2x]$$

$$= \frac{1}{2} \left[\frac{\sin 4x}{4} + \frac{\sin 2x}{2} \right] + C$$

$$\textcircled{3} \int \cos x / \sin^2 x \, dx$$

$$\text{Let } u = \sin^2 x$$

$$\frac{du}{dx} = 2 \cos x \Rightarrow dx = \frac{du}{2 \cos x}$$

$$\int \frac{\cos x}{u^2} \cdot \frac{du}{2 \cos x} = \int \frac{1}{2u^2} du$$

$$= \int \frac{1}{u^2} du = \left[\frac{u^{-2+1}}{-2+1} \right]$$

$$= \frac{1}{u} = \frac{1}{\sin x} + C$$

$\textcircled{4}$ Double integral with limits from 1 to 3, from 0 to 3
 $(9x^2y) \, dx \, dy$

Solution

$$\int_1^3 \int_0^3 9x^2y \, dx \, dy = \frac{243}{2}$$