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Questions

Find the integral of the following

1). Sin7xCos2x dx

2). Cos3xCos$x$ dx

3). $\frac{\cos x}{sin^2 x\ dx}$

4). Double integral with limits from 1 to 2, from 0 to 3 $(9x^2y)\frac{dx}{dy}$

SOLUTION

1. Find the integral of Sin7xCos2xdx
   $\int \cos(2x)\sin(7x)dx$
   Apply product-to-sum formulas:
   $\sin(x)\sin(y) = \frac{1}{2}(\cos(y - x) - \cos(y + x))$, $\sin2(x) = \frac{1}{2}(1 - \cos(2x))$,
   $\cos(x)\cos(y) = \frac{1}{2}(\cos(y + x) + \cos(y - x))$, $\cos2(x) = \frac{1}{2}(\cos(2x) + 1)$,
   $\sin(x)\cos(y) = \frac{1}{2}(\sin(y + x) - \sin(y - x))$, $\cos(x)\sin(x) = \frac{1}{2}\sin(2x)$

   $= \int \frac{\sin(9x) + \sin(5x)}{2}dx$
   Apply linearity:
   $= \frac{1}{2}\int \sin(9x)dx + \frac{1}{2}\int \sin(5x)dx$
   Now solving:
   $\int \sin(9x)dx$
   Substitute $u = 9x \longrightarrow \frac{du}{dx} = 9$
   $\longrightarrow dx = \frac{1}{9}du$
   $= \frac{1}{9}\int \sin(u)du$
   Now solving:
   $\int \sin(u)du$
   This is a standard integral:
   $= -\cos(u)$
   Plug in solved integrals:
   $\frac{1}{9}\int \sin(u)du$

$= -\frac{\cos(u)}{9}$
Undo substitution $u = 9x$
$= -\frac{\cos(9x)}{9}$
Now solving:
$\int \sin(5x)dx$
Substitute $u = 5x \longrightarrow \frac{du}{dx} = 5$
$\longrightarrow dx = \frac{1}{5}\, du$
$= \frac{1}{5} \int \sin(u)du$
Now solving:
$\int \sin(u)\, du$
Use previous result:
$= -\cos(u)$
Plug in solved integrals:
$\frac{1}{5} \int \sin(u)\underline{du}$
$= -\frac{\cos(u)}{5}$
Undo substitution $u = 5x$:
$= -\frac{\cos(5x)}{5}$
Plug in solved integrals:
$= \frac{1}{2} \int \sin(9x)dx + \frac{1}{2} \int \sin(5x)dx$
$= \frac{1}{2} \times -\frac{\cos(9x)}{9} + \frac{1}{2} \times -\frac{\cos(5x)}{5}$
$= -\frac{\cos(9x)}{18} - \frac{\cos(5x)}{10}$
The problem is solved:
$\int \cos(2x)\sin(7x)\, dx$
$= -\frac{\cos(9x)}{18} - \frac{\cos(5x)}{10} + C$

2. Find the integral of Cos3xCos$x$ dx
$\int \cos(x)\cos(3x)\, dx$
Write $\cos(3x)$ as $\cos(x)\cos(2x) - \sin(x)\sin(2x)$
using the formula $\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$:
$= \int \cos(x)(\cos(x)\cos(2x) - \sin(x)\sin(2x))\, dx$
Expand:
$= \int (cos^2(x)\cos(2x) - \cos(x)\sin(x)\sin(2x))\, dx$
Apply linearity:
$= \int cos^2(x)\cos(2x)\, dx - \int \cos(x)\sin(x)\sin(2x)\, dx$

Now solving:
$\int cos^2(x)\cos(2x)\, dx$
Apply product-to-sum formulas:

$\sin(x)\sin(y) = \frac{1}{2}(\cos(y - x) - \cos(y + x))$, $\sin2(x) = \frac{1}{2}(1 - \cos(2x))$,
$\cos(x)\cos(y) = \frac{1}{2}(\cos(y + x) + \cos(y - x))$, $\cos2(x) = \frac{1}{2}(\cos(2x) + 1)$,
$\sin(x)\cos(y) = \frac{1}{2}(\sin(y + x) - \sin(y - x))$, $\cos(x)\sin(x) = \frac{1}{2}\sin(2x)$
$= \int (\frac{\cos(4x) + 2\cos(2x)}{4} + \frac{1}{4})\, dx$
Apply linearity:
$= \frac{1}{4}\int \cos(4x)\, dx + \frac{1}{2}\int\cos(2x)\, dx + \frac{1}{4}\int 1dx$

Now solving:
$\int \cos(4x)\, dx$
Substitute $u = 4x \longrightarrow \frac{du}{dx} = 4$
$\longrightarrow dx = \frac{1}{4}du$
$= \frac{1}{4}\int \cos(u)\, du$

Now solving:
$\int \cos(u)\, du$
This is a standard integral:
$= \sin(u)$

Plug in solved integrals:
$\frac{1}{4}\int \cos(u)\, du$
$= \frac{\sin(u)}{4}$
Undo substitution $u = 4x$:
$= \frac{\sin(4x)}{4}$

Now solving:
$\int \cos(2x)\, dx$
Substitute $u = 2x \longrightarrow \frac{du}{dx} = 2$
$\longrightarrow dx = \frac{1}{2}du$
$= \frac{1}{2}\int \cos(u)\, du$

Now solving:
$\int \cos(u)\, du$
Use previous result:
$= \sin(u)$
Plug in solved integrals:
$\frac{1}{2}\int \cos(u)\, du$
$= \frac{\sin(u)}{2}$
Undo substitution $u = 2x$:

$= \frac{\sin(2x)}{2}$

Now solving:

$\int 1 \mathrm{d}x$

Apply constant rule:

$= x$

Plug in solved integrals:

$\frac{1}{4} \int \cos(4x)\, \mathrm{d}x + \frac{1}{2} \int \cos(2x)\, \mathrm{d}x + \frac{1}{4} \int 1 \mathrm{d}x$

$= \frac{\sin(4x)}{16} + \frac{\sin(2x)}{4} + \frac{x}{4}$

Now solving:

$\int \cos(x)\sin(x)\sin(2x)\, \mathrm{d}x$

Apply product-to-sum formulas:

$\sin(x)\sin(y) = \frac{1}{2}(\cos(y - x) - \cos(y + x))$, $\sin2(x) = \frac{1}{2}(1 - \cos(2x))$,

$\cos(x)\cos(y) = \frac{1}{2}(\cos(y + x) + \cos(y - x))$, $\cos2(x) = \frac{1}{2}(\cos(2x) + 1)$,

$\sin(x)\cos(y) = \frac{1}{2}(\sin(y + x) - \sin(y - x))$, $\cos(x)\sin(x) = \frac{1}{2}\sin(2x)$

$= \int (\frac{1}{4} - \frac{\cos(4x)}{4})\, \mathrm{d}x$

Apply linearity:

$= \frac{1}{4} \int 1 \mathrm{d}x - \frac{1}{4} \int \cos(4x)\, \mathrm{d}x$

Now solving:

$\int 1 \mathrm{d}x$

Use previous result:

$= x$

Now solving:

$\int \cos(4x)\, \mathrm{d}x$

Use previous result:

$= \frac{\sin(4x)}{4}$

Plug in solved integrals:

$\frac{1}{4} \int 1 \mathrm{d}x - \frac{1}{4} \int \cos(4x)\, \mathrm{d}x$

$= \frac{x}{4} - \frac{\sin(4x)}{16}$

Plug in solved integrals:

$\int cos^2(x)\cos(2x)\, \mathrm{d}x - \int \cos(x)\sin(x)\sin(2x)\, \mathrm{d}x$

$= \frac{\sin(4x)}{8} + \frac{\sin(2x)}{4}$

The problem is solved:

$\int \cos(x)\cos(3x)\, \mathrm{d}x$

$= \frac{\sin(4x)}{8} + \frac{\sin(2x)}{4} + C$

3. Find the integral of $\frac{\cos x}{sin^2 x\ dx}$

$\int \frac{\cos x}{sin^2 x\ dx}$ dx

Substitute u = sin(x) ⟶ $\frac{du}{dx}$ = cos(x)

⟶ dx = $\frac{1}{\cos(x)}$ du

= $\int \frac{1}{u^2}$ du

Apply power rule:

$\int u^n$ du = $\frac{u^{n+1}}{\mathrm{n}+1}$ with $n = -2$

$= -\frac{1}{u}$

Undo substitution u = sin(x)

$= -\frac{1}{\sin(x)}$

The problem is solved:

$\int \frac{\cos x}{sin^2 x\ dx}$ dx

$= -\frac{1}{\sin(x)}$ + C

4. Find the double integral with limits from 1 to 2, from 0 to 3 $(9x^2y)\frac{dx}{dy}$

$\int 9x^2y$ dy

Apply linearity:

= $9x^2 \cdot \int y$ dy

Now solving:

$\int y$ dy

Apply power rule:

$\int y^n$ dy = $\frac{y^{n+1}}{n+1}$ with n = 1

$= \frac{y^2}{2}$

Plug in solved integrals:

$9x^2 \cdot \int y$ dy

$= \frac{9x^2y^2}{2}$

The problem is solved:

$\int 9x^2y$ dy

$= \frac{9x^2y^2}{2}$ + C

$\therefore \int_1^2 f_{(y)}\, \mathrm{d}y = \frac{27x^2}{2}$

Second derivative

$\int \frac{9x^2y^2}{2}\, \mathrm{d}y$

Apply linearity:

$= \frac{9x^2}{2} \int y^2\, \mathrm{d}y$

Now solving:

$\int y^2 \mathrm{d}y$

Apply power rule:

$\int y^n\, \mathrm{d}y = \frac{y^{n+1}}{n+1}$ with n = 2

$= \frac{y^3}{3}$

Plug in solved integrals:

$\frac{9x^2}{2} \int y^2\, \mathrm{d}y$

$= \frac{3x^2y^3}{2}$

The problem is solved:

$\int \frac{9x^2y^2}{2}\, \mathrm{d}y$

$= \frac{3x^2y^3}{2} + C$

$\therefore \int_0^3 f_{(y)}\, \mathrm{d}y = \frac{81x^2}{2}$