

1) $\int \sin 7x \cos 2x$

Let $u = \cos 2x$, $du = -2 \sin 2x$

$du = -2 \sin 2x dx$, $v = -\frac{\cos 7x}{7}$

$uv - \int v du$

$-\frac{\cos 2x \cos 7x}{7} - \int \frac{2 \cos 7x \sin 2x}{7} dx$

$-\frac{\cos 2x \cos 7x}{7} - \frac{2}{7} \int \cos 7x \sin 2x dx$

Let $u = \sin 2x$, $du = 2 \cos 2x$

$du = 2 \cos 2x$, $v = \frac{\sin 7x}{7}$

$uv - \int v du$

$-\frac{\cos 2x \cos 7x}{7} - \frac{2}{7} \left(\frac{\sin 2x \sin 7x}{7} - \int \frac{2 \sin 7x \cos 2x}{7} dx \right)$

$-\frac{\cos 2x \cos 7x}{7} - \frac{2}{7} \left(\frac{\sin 2x \sin 7x}{7} - \frac{2}{7} \int \sin 7x \cos 2x dx \right)$

$= \int \sin 7x \cos 2x dx = -\frac{\cos 2x \cos 7x}{7} - \frac{2}{7} \left(\frac{\sin 2x \sin 7x}{7} \right)$

$- \frac{2}{7} \int \sin 7x \cos 2x dx$

$= -\frac{49 \cos 2x \cos 7x - 14 \sin 4x \sin 7x}{315} + C$

$$2 \int \cos 3x \cos x \, dx$$

$$u = \cos x$$

$$du = -\sin x \, dx$$

$$du = -\sin x \, dx$$

$$v = \frac{\sin 3x}{3}$$

$$uv - \int v \, du$$

$$\frac{\cos x \sin 3x}{3} - \int -\frac{\sin 3x \sin x}{3} \, dx$$

$$\frac{\cos x \sin 3x}{3} + \frac{1}{3} \int \sin 3x \sin x \, dx$$

$$\text{Let } u = \sin x$$

$$du = \cos x \, dx$$

$$du = \cos x \, dx$$

$$v = -\frac{\cos 3x}{3}$$

$$uv - \int v \, du$$

$$\frac{\cos x \sin 3x}{3} + \frac{1}{3} \left(-\frac{\sin x \cos 3x}{3} - \int -\frac{\cos 3x \cos x}{3} \, dx \right)$$

$$\frac{\cos x \sin 3x}{3} + \frac{1}{3} \left(-\frac{\sin x \cos 3x}{3} + \frac{1}{3} \int \cos 3x \cos x \, dx \right)$$

$$\int \cos 3x \cos x \, dx = \frac{\cos x \sin 3x}{3} + \frac{1}{3} \left(-\frac{\sin x \cos 3x}{3} \right)$$

$$+ \frac{1}{3} \int \cos 3x \cos x \, dx$$

$$= \frac{3 \cos x \sin 3x - \sin x \cos 3x}{8} + C$$