

$$5) \int \cos 5x \cos 6x \, dx = \frac{1}{2} (\cos(A+B) + \cos(A-B))$$

$$A = 5x, B = 6x$$

$$= \frac{1}{2} (\cos 11x + \cos -x)$$

$$= \frac{1}{2} (\cos 11x - \cos x)$$

$$= \frac{1}{2} \int (\cos 11x - \cos x) \, dx$$

$$= \frac{1}{2} (\sin 11x - \sin x) + C$$

$$= \frac{1}{2} \left(\frac{\sin 11x}{11} - \frac{\sin x}{1} \right) + C$$

$$= \frac{\sin 11x}{22} - \frac{\sin x}{2} + C$$

$$3) \int x^2 \sin x dx$$

$$u = x^2 \quad du = 2x dx$$

$$\frac{du}{dx} = \frac{2x^3}{3}$$

$$du = \frac{2x^3}{3} dx \quad v = -\cos x$$

$$\int u dv = uv - \int v du$$

$$= x^2 \cdot (-\cos x) - \int (-\cos x) \cdot \frac{2x^3}{3} dx$$

$$= -\cos x (x^2) - (-\sin x) \cdot \frac{x^4}{12} + C$$

$$\int x^2 \sin x dx = -\cos x (x^2) + \sin x \left(\frac{x^4}{12} \right) + C$$

$$4) \sin 7x \cos 2x = \frac{1}{2} (\sin(A+B) + \sin(A-B))$$

$$A = 7x \quad B = 2x$$

$$= \frac{1}{2} (\sin 9x + \sin 5x)$$

$$\frac{1}{2} (\int \sin 9x + \sin 5x) dx$$

$$\frac{1}{2} (\cos 9x + \cos 5x) dx$$

$$\frac{1}{2} \left(\frac{\cos 9x}{9} + \frac{\cos 5x}{5} \right) + C$$

$$\sin 7x \cos 2x = \frac{\cos 9x}{18} + \frac{\cos 5x}{5} + C$$

ÜBUNG 11: PARTIELLE INTEGRATION

1) $\int \ln x \, dx$

1) $\int 2x^2 \ln x \, dx$

$$u = \ln x \quad dv = 2x^2$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x} dx \quad v = \frac{2x^3}{3}$$

$$\begin{aligned} \int u dv &= uv - \int v du \\ &= \ln x \left(\frac{2x^3}{3} \right) - \int \frac{2x^3}{3} \cdot \frac{1}{x} dx \\ &= \frac{2x^3 \ln x}{3} - \int \frac{2x^2}{3} dx \\ &= \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + C \end{aligned}$$

$$\int 2x^2 \ln x \, dx = \frac{2x^3}{3} \left[\ln x - \frac{1}{3} \right] + C$$

2) $\int 3t e^{2t} dt$

$$u = 3t \quad dv = e^{2t}$$

$$\frac{du}{dt} = \frac{3t^2}{2} \quad v = \frac{1}{2} e^{2t}$$

$$du = \frac{3t^2}{2} dt$$

$$\begin{aligned} \int u dv &= uv - \int v du \\ &= 3t \left(\frac{1}{2} e^{2t} \right) - \int \frac{1}{2} e^{2t} \cdot \frac{3t^2}{2} dt \end{aligned}$$

$$= \frac{3t e^{2t}}{2} - \frac{3t^2 e^{2t}}{4}$$

$$\int 3t e^{2t} dt = \frac{3t e^{2t}}{2} - \frac{3t^2 e^{2t}}{4} + C$$

$$\int 3t e^{2t} dt = \frac{3t e^{2t}}{2} - \frac{3t^2 e^{2t}}{4} + C$$