

1. $2x^2 \ln x$

$u = \ln x$

$dv = 2x^2$

$du = \frac{1}{x}$

$v = \frac{2x^3}{3}$

$$\int u dv = uv - \int v du$$

$$\int u dv = \ln x \times \frac{2x^3}{3} - \int \frac{2x^3}{3} \times \frac{1}{x} dx$$

$$\int u dv = \frac{2x^3 \ln x}{3} - \int \frac{2x^2}{3}$$

$$\int u dv = \frac{2x^3 \ln x}{3} - \frac{2x^3}{9}$$

$$\int u dv = \frac{2x^3 (\ln x - 1)}{3} + C$$

2. $3te^{2t}$

$u = 3t$

$v = e^{2t}$

$du = 3$

$v = e^{2t}/2$

$$\int u dv = uv - \int v du$$

$$\int u dv = 3t \times \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \times 3$$

$$\int u dv = \frac{3te^{2t}}{2} - \frac{e^{2t}}{2} \times \frac{3}{2}$$

$$\int u dv = \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4}$$

$$\therefore \int u dv = \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$$

$$3. \quad x^2 \sin x$$

$$u = x^2 \quad dv = \sin x$$

$$du = 2x dx \quad v = -\cos x$$

$$\begin{aligned} \int u dv &= uv - \int v du \\ &= x^2 x - \cos x + \int -\cos x \times 2x dx \\ &= -x^2 \cos x + \int \cos x \times 2x dx \\ &= -x^2 \cos x + 2 \int \cos x \times x dx \quad \dots \text{equ (i)} \end{aligned}$$

$$\text{Let } u = x \quad dv = \cos x$$

$$du = dx \quad v = \sin x$$

$$\begin{aligned} \int u dv &= uv - \int v du \\ &= x \times \sin x - \int \sin x dx \quad \dots \text{equ (ii)} \end{aligned}$$

Sub equ (ii) into equ (i)

$$\int u dv = -x^2 \cos x + 2(x \sin x - \int \sin x dx)$$

$$\int u dv = -x^2 \cos x + 2x \sin x - 2(-\cos x) + C$$

$$\int u dv = -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$4. \cos 5x \cos 6x =$$

$$= \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$= \frac{1}{2} [\cos(5x+6x) + \cos(5x-6x)]$$

$$= \frac{1}{2} [\cos 11x - \cos x]$$

$$= \frac{1}{2} [\sin 11x - (+ \sin x)]$$

$$= \frac{1}{2} + \frac{\sin 11x}{2} - \frac{\sin x}{2}$$

$$\cos 5x \cos 6x = \frac{1}{2} + \frac{\sin 11x}{2} - \frac{\sin x}{2} \therefore \frac{\sin 11x}{2} - \frac{\sin x}{2} + C$$

$$5. \sin 7x \cos 2x$$

$$= \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$= \frac{1}{2} [-\cos 9x - \cos 5x]$$

$$= \frac{1}{2} [\sin(7x+2x) + \sin(7x-2x)]$$

$$= \frac{-\cos 9x}{18} - \frac{\cos 5x}{10}$$

$$= \frac{1}{2} [\sin 9x + \sin 5x]$$

$$\sin 7x \cos 2x = \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$$