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19/MH/01/286

MBBS

Question

Integrate the following functions

1) $2x^2 \ln x$

Using Integration by Parts formula

$\int u\,\delta v = uv - \int v\,\delta u$

$u = \ln x \quad \frac{\delta u}{\delta x} = \frac{1}{x} \quad \delta u = \frac{\delta x}{x}$

$\delta v = 2x^2$

$v = \frac{2x^3}{3}$

$\int \ln x\, 2x^2 = \ln x . \frac{2x^3}{3} - \int \frac{2x^3}{3} . \frac{\delta x}{x}$

$= \ln x . \frac{2x^3}{3} - \frac{2}{3}\int x^2 . \delta x.$

$= \ln x . \frac{2x^3}{3} - \frac{2}{3} . \frac{x^3}{3} + C$

$\int \ln x\, 2x^2 = \ln x . \frac{2x^3}{3} - \frac{2x^3}{9} + C$ //ANS

2) $3te^{2t}$

$\int u\,\delta v = uv - \int v\,\delta u$

$u = 3t \quad \frac{\delta u}{\delta x} = 3 \quad \delta u = 3\,\delta x$

$\delta v = e^{2t}$

$v = \frac{1}{2} . e^{2t}$

$\int 3te^{2t} = 3t . \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} . 3\delta x.$

$= 3t . \frac{e^{2t}}{2} - \frac{3}{2}\int e^{2t} . \delta x$

$= 3t . \frac{e^{2t}}{2} - \frac{3}{2} . \frac{e^{2t}}{2} + C$

$\int 3te^{2t} = 3t . \frac{e^{2t}}{2} - \frac{3e^{2t}}{4} + C$ //ANS

3) $x^2 \sin x$

$\int u\,\delta v = uv - \int v\,\delta u$

$u = x^2 \quad \frac{\delta u}{\delta x} = 2x \quad \delta u = 2x\,\delta x.$

$\delta v = \sin x$

$v = -\cos x$

$\int x^2 \sin x = x^2 . -\cos x - \int -\cos x . 2x\,\delta x$

$= -x^2\cos x + \int \cos x . 2x\,\delta x$

using

$\int \cos x . 2x$

$u = \cos x, \quad \frac{\delta u}{\delta x} = -\sin x \quad \delta u = -\sin x\,\delta x$

$\delta v = 2x$

$v = 2x^2/2$

$\int \cos x . 2x = \cos x \frac{2x^2}{2} - \int \frac{2x^2}{2} . -\sin x\,\delta x$

$= \cos x \frac{2x^2}{2} + \int \frac{2x^2}{2} \sin x\,\delta x$

$= \cos x \frac{2x^2}{2} + \int x^2 \sin x\,\delta x \ldots$

4)

~~Using $\int x^2 \sin x$~~

$$\int x^2 \sin x = -x^2\cos x + 2\int \cos x \cdot x \,\delta x \ldots\ldots *$$

Using

$\int \cos x \cdot x \,\delta x$

$u = x \qquad \delta v = \cos x$

$\frac{\delta u}{\delta x} = 1 \qquad v = \sin x$

$\delta u = \delta x$

$$\int \cos x \cdot x\,\delta x = x \cdot \sin x - \int \sin x \cdot \delta x$$

$$= x\sin x - (-\cos x) + C$$

$$\int \cos x \cdot x\,\delta x = x\sin x + \cos x + C \ldots *$$

$\therefore$

$$\int x^2 \sin x = -x^2\cos x + 2x\sin x + 2\cos x + C$$

//ANS

4) $\int \cos 5x \cos 6x \,\delta x$

Solu

$$\int \cos 5x \cos 6x \,\delta x = \frac{1}{2}\int \cos(5x+6x) + \cos(5x-6x)$$

$$= \frac{1}{2}\int \cos 11x - \cos x$$

$$= \frac{1}{2}\left[\frac{\sin 11x}{11} - \frac{\sin x}{1}\right] + C$$

$$\int \cos 5x \cdot \cos 6x\,\delta x = \frac{\sin 11x}{11} - \frac{\sin x}{2} + C$$

//ANS

5) $\int \sin 7x \cos 2x \,\delta x$

Solu

$$\int \sin 7x \cos 2x\,\delta x = \frac{1}{2}\int \sin(9x) + \sin 5x$$

$$= \frac{1}{2}\left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right] + C$$

$$= \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$$

$$\int \sin 7x \cos 2x\,\delta x = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$

//ANS