Omotayo Miracle
MBBS
19/MHS01/345
Maths 104

(1) $2x^2 \ln x$

Solution:

$u = \ln x \qquad dv = 2x^2$

$\frac{du}{dx} = \frac{1}{x} dx \qquad V = \frac{2x^3}{3}$

$\int x dv = uv - \int v du$

$= \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{dx}{x}$

$= \frac{2x^3}{3} \ln x - \int \frac{2x^2}{3} dx$

$= \frac{2x^3}{3} \ln x - \frac{2x^3}{3 \times 3} + c$

$\therefore \int 2x^2 \ln x dx = \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + c$

(2) $3te^{2t}$

Solution:

$U = 3t \qquad dv = e^{2t}$

$du = 3dx \qquad V = \frac{1}{2} e^{2t}$

$\int u dv = uv - \int v du$

$\int 3te^{2t} = 3t \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \cdot 3dx$

$\int 3te^{2t} = \frac{3t}{2} e^{2t} - \int \frac{3}{2} e^{2t} \cdot dx$

$= \frac{3t}{2} e^{2t} - \frac{1}{2} \cdot \frac{3}{2} e^{2t} + c$

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$$\therefore \int 3te^{2t}\,dt = \left[\frac{3te^{2t}}{2} - \frac{3}{4}e^{2t}\right] + c$$

(3) $x^2 \sin x$

Solution:

$u = x^2 \qquad dv = \sin x$

$\frac{du}{dx} = 2x \qquad v = -\cos x$

$du = 2x\,dx$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 \sin x\,dx = x^2 - \cos x - \int -\cos x \cdot 2x\,dx$$

$$= -x^2\cos x + \int 2x\cos x\,dx$$

$$= -x^2\cos x + \left[\begin{matrix} u = 2x & dv = \cos x \\ du = 2\,dx & v = \sin x \end{matrix}\right]$$

$$= -x^2\cos x + uv - \int v\,du$$

$$= -x^2\cos x + 2x\sin x - \int \sin x \cdot 2\,dx$$

$$\int x^2 \sin x = -x^2\cos x + 2x\sin x - \int 2\sin x$$

$$\therefore \int x^2 \sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + c$$

(4) $\cos 5x \cos 6x$

Solution:

$A = 5x, \quad B = 6x$

Recall that:

$$\cos A \cos B = \tfrac{1}{2}\left[\cos(A+B) - \cos(A-B)\right)$$

$$= \tfrac{1}{2}\left[\cos 11x + \cos x\right.$$

$$\int \cos 5x \cos 6x\,dx = \tfrac{1}{2}\left[\frac{\sin 11x}{11} + \frac{\sin x}{\cdot}\right] + c$$

$$\therefore \int \cos 5x \cos 6x\,dx = \frac{\sin 11x}{22} + \frac{\sin x}{2} + c$$

(5) $\sin 7x \cos 2x$

Solutions

$A = 7x,\ B = 2x$

Recall that;

$$\sin A \cos B = \tfrac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\int \sin 7x \cos 2x \, dx = \tfrac{1}{2}\int \sin 9x + \sin 5x$$

$$\int \sin 7x \cos 2x \, dx = \tfrac{1}{2}\left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5}\right] + c$$

$$\therefore \int \sin 7x \cos 2x \, dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + c_1$$