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MBBS

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1.) $2x^2 \ln x$

solution

$u = \ln x \qquad dv = 2x^2$

$\frac{du}{dx} = \frac{1}{x} dx \qquad v = \frac{2x^3}{3}$

$\int u\,dv = uv - \int v\,du$

$= \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{dx}{x}$

$= \frac{2x^3}{3} \ln x - \int \frac{2x^2}{3} dx$

$= \frac{2x^3}{3} \ln x - \frac{2x^3}{3 \times 3} + C$

$\therefore \int 2x^2 \ln x\,dx = \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + C$

OR

$\frac{2}{3} x^3 \left(\ln x - \frac{1}{3}\right) + C$ //

2.) $3te^{2t}$

solution;

$u = 3t \qquad dv = e^{2t}$

$du = 3dx \qquad v = \frac{1}{2} e^{2t}$

$\int u\,dv = uv - \int v\,du$

$\int 3te^{2t} = 3t \cdot \frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t} \cdot 3\,dx$

$\int 3te^{2t} = \frac{3t}{2}e^{2t} - \int \frac{3}{2}e^{2t}\,dx$

$= \frac{3te^{2t}}{2} - \frac{1}{2} \cdot \frac{3}{2}e^{2t} + c$

$\therefore \int 3te^{2t}\,dt = \left[\frac{3te^{2t}}{2} - \frac{3}{4}e^{2t}\right] + c$

3) $x^2 \sin x$

solution

$u = x^2$ $\quad dv = \sin x$

$\frac{du}{dx} = 2x$ $\quad v = -\cos x$

$dx = 2x\,dx$

$\int u\,dv = uv - \int v\,du$

$\int x^2 \sin x\,dx = +x^2 - \cos x - \int -\cos x \cdot 2x\,dx$

$= -x^2 \cos x + \int 2x \cos x\,dx$

$= -x^2 \cos x + \left[\begin{array}{ll} u = 2x & dv = \cos x \\ du = 2dx & v = \sin x \end{array}\right]$

$-x^2 \cos x + uv - \int v\,du$

$-x^2 \cos x + 2x \sin x - \int \sin x \cdot 2\,dx$

$\int x^2 \sin x = -x^2 \cos x + 2x \sin x - \int 2 \sin x$

$\therefore \int x^2 \sin x\, dx = -x^2 \cos x + 2x \sin x + 2\cos x + C$

4) $\cos 5x \cos 6x$

solution

$A = 5x$, $B = 6x$

Recall that;

$\cos A \cos B = \frac{1}{2}[\cos(A+B) - \cos(A-B)]$

$= \frac{1}{2}\int \cos 11x + \cos x$

$\int \cos 5x \cos 6x\, dx = \frac{1}{2}\left[\frac{\sin 11x}{11} + \sin x\right] + C$

$\therefore \int \cos 5x \cos 6x\, dx = \frac{\sin 11x}{22} + \frac{\sin x}{2} + C$

5.) $\sin 7x \cos 2x$

solution;

$A = 7x$, $B = 2x$

Recall that;

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\int \sin 7x \cos 2x\, dx = \frac{1}{2}\int \sin(A+B) + \sin(A-B)]$

$$\therefore \int \sin 7x \cos 2x \, dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$